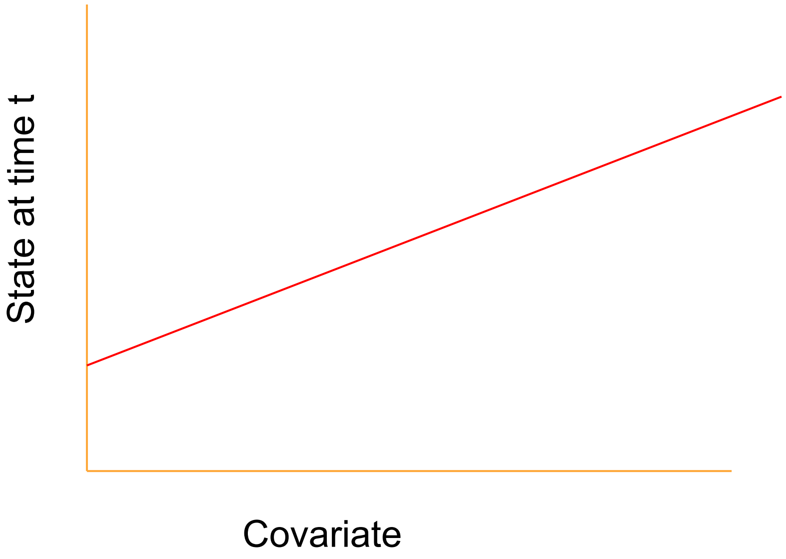
Dynamic Models

A Primer

Outline

- What is a Dynamic model (and how it’s not a regression model)
- Examples
  - X
  - rho*X
  - B0 + rho*X
  - rho*X + B2*X^2
  - X + B1*Z
  - X + B1*dZ
  - K = b0 + b1*Z
- Thoughts on how to build models
  - A simple ecosystem model

What is a dynamic model?



This is not a dynamic model

Directly relates x to y

Forecasting with this equation is easy

y = mx + b + N(0, sigma)

Where you sample from the uncertainty in
M and b: parameters (see note 1)
X: covariate or driver
N(0, sigma): ”process” (see note 2

Note1:Technically: sigma is also a parameter with uncertainty
Note2: N(0, sigma) combines errors in model structure with uncertainty in observations

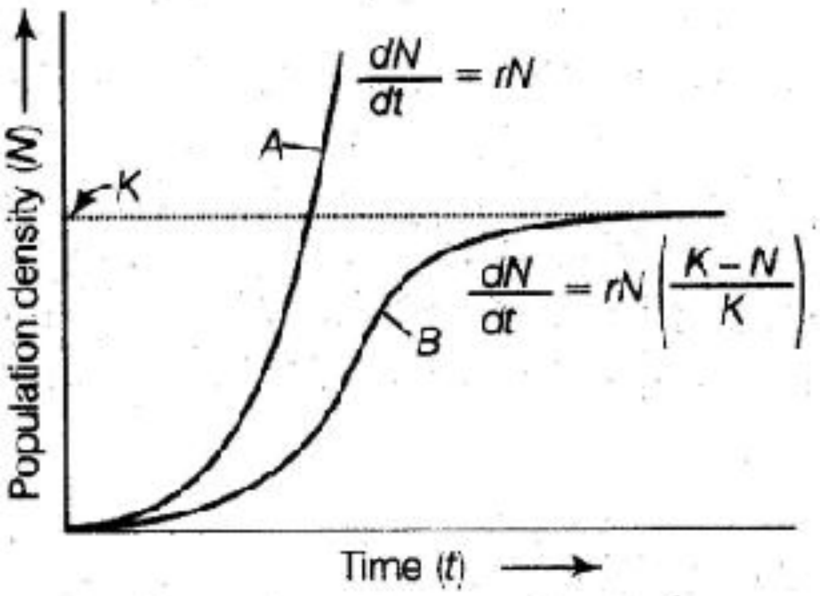
What is a dynamic model?

Models explaining **change over time**

Models where the future state is a **function of the current state** (and lags)

Dynamic models force us to focus on the PROCESS that causes CHANGE.

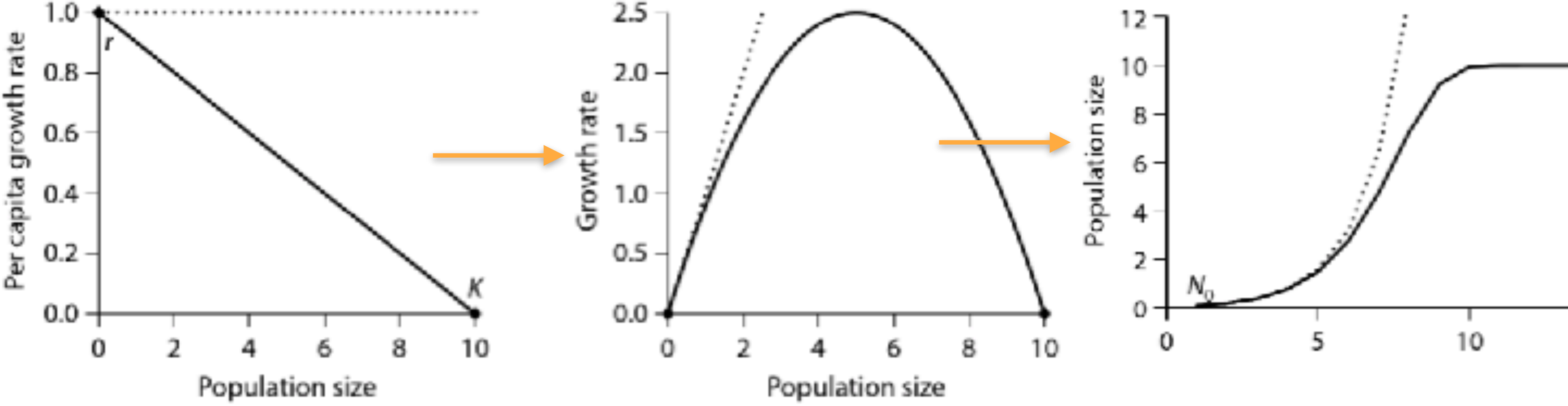
Behave differently from regression



$$N_{t+1} = N_t + rN\left(1 - \frac{N}{K}\right)$$

Previous state

Slope (change)

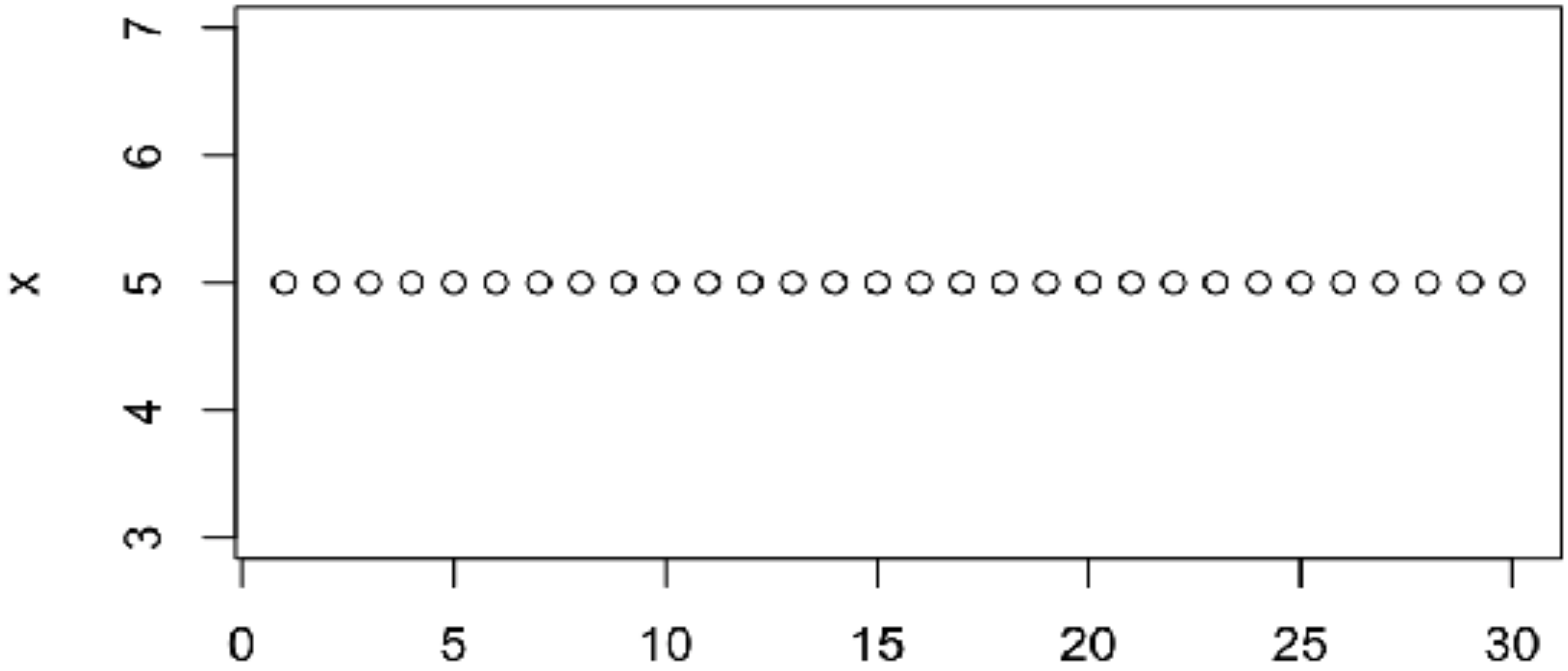


Principle: Start simple, add complexity incrementally

$x[t+1] = x[t]$

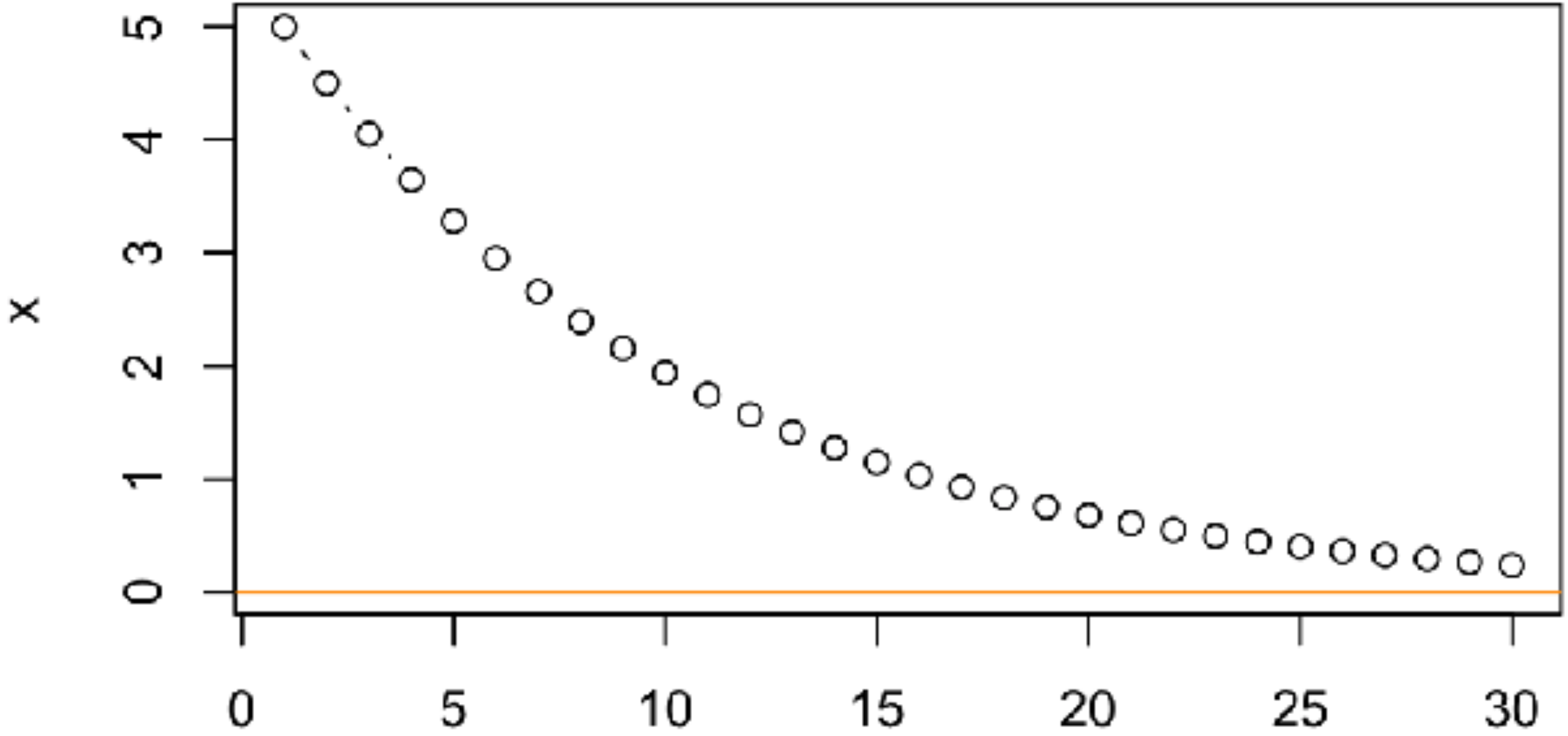
Persistence forecast / random walk

Uninteresting, but useful NULL



x[t+1] = rho*x[t]

exponential growth / decline
autoregressive (AR1)

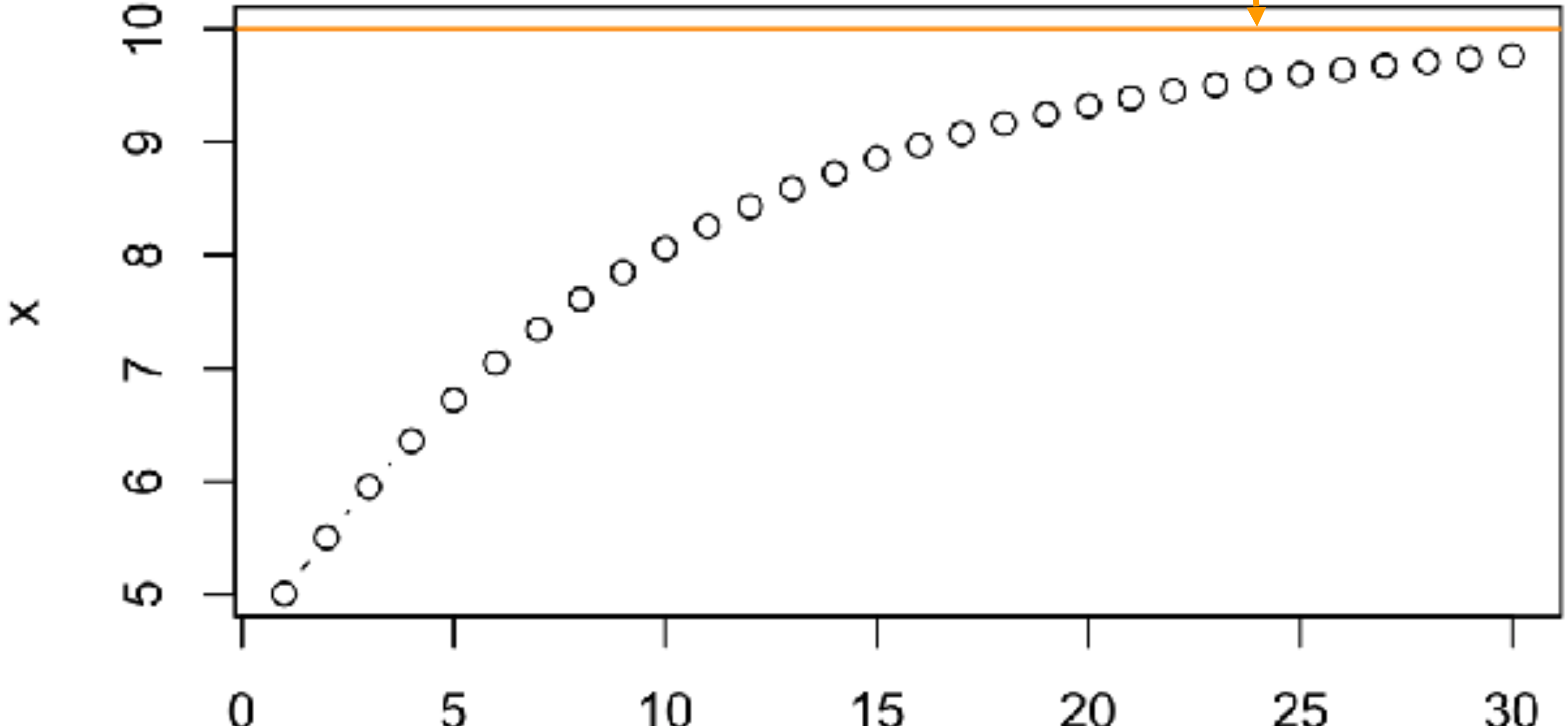


$x[t+1] = b0 + rho*x[t]$

Has a non-zero equilibrium

$X = b0/(1-rho)$

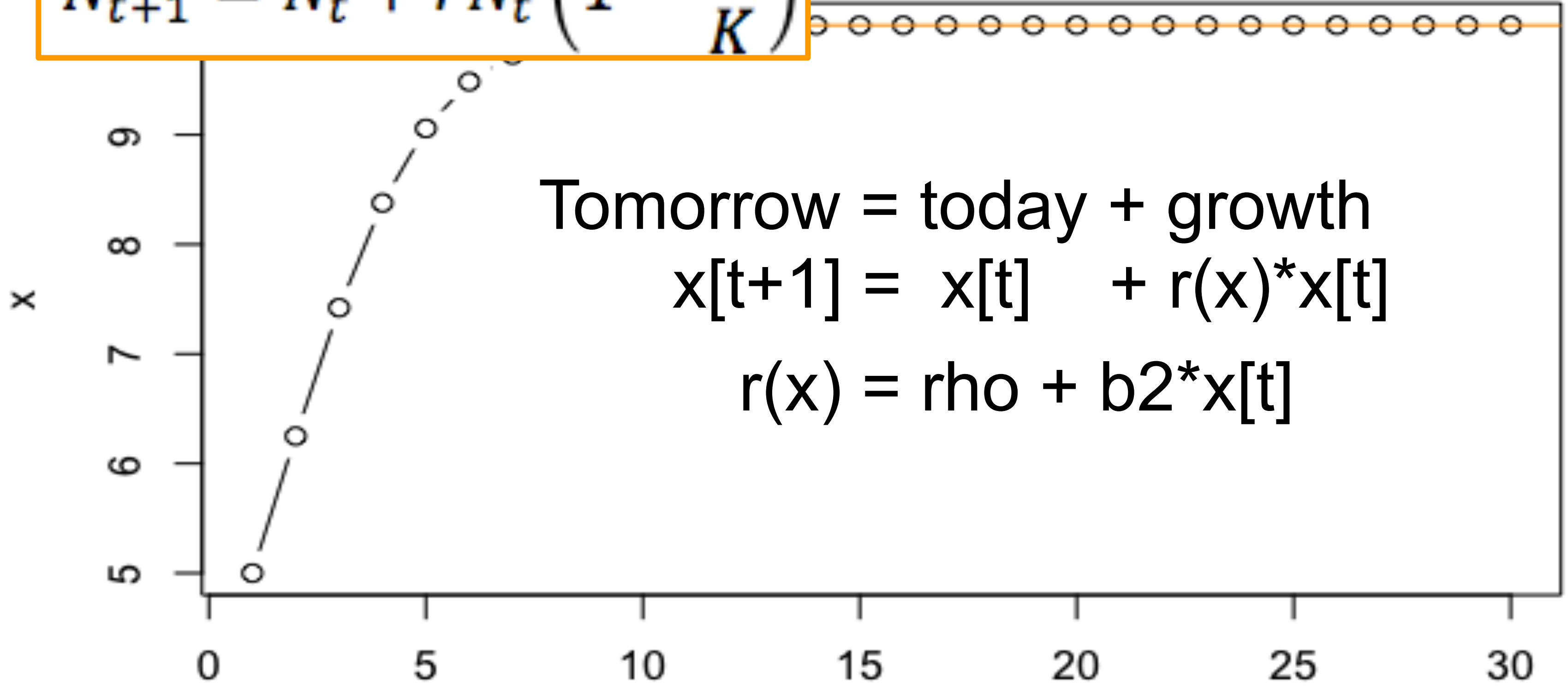
Density Independent (DI) growth trend



logistic: rho = (1+r)
b2 = -r/K

x[t+1] = rho*x[t] + b2*x[t]^2

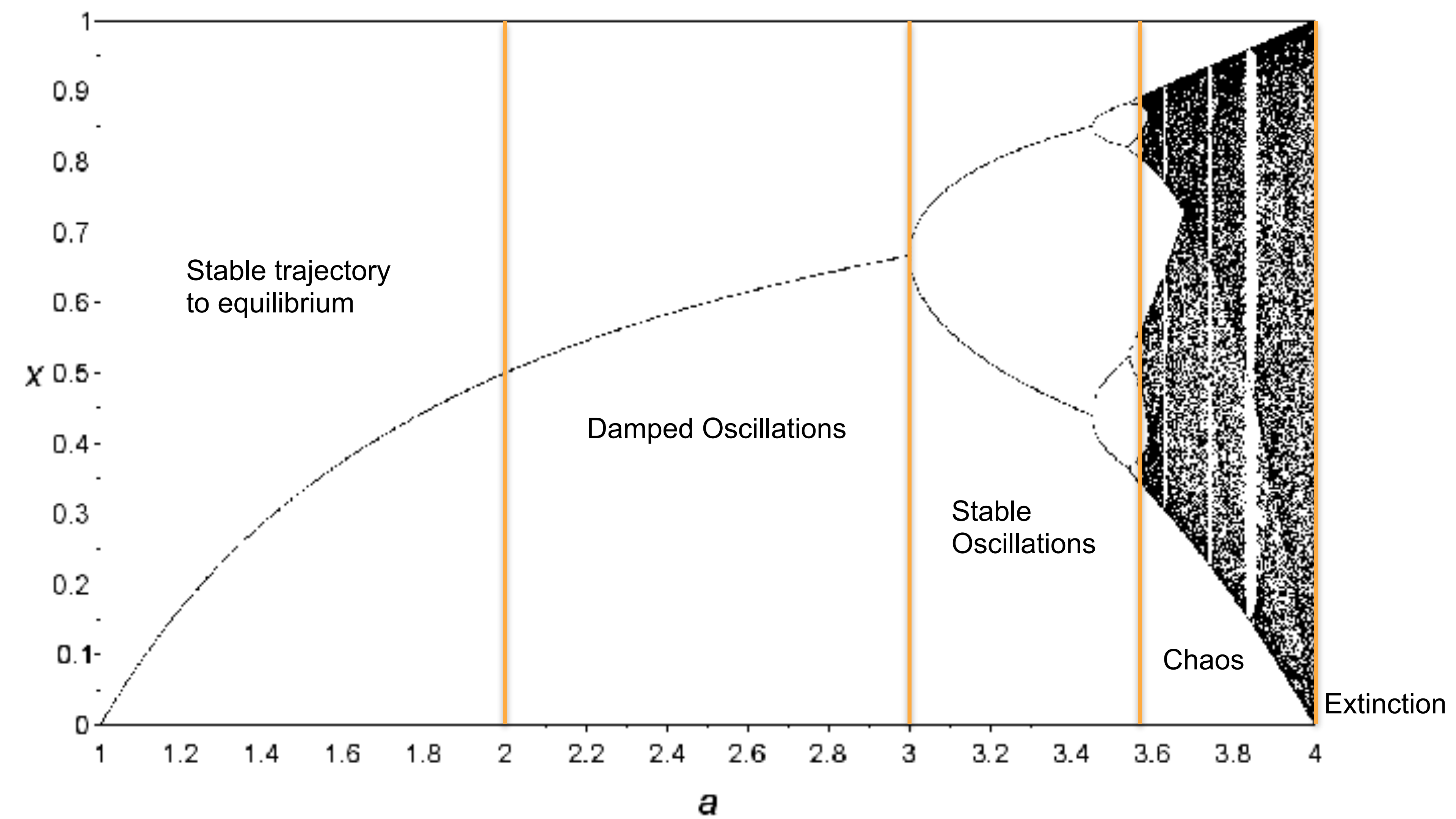
$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right)$$



Tomorrow = today + growth

x[t+1] = x[t] + r(x)*x[t]

r(x) = rho + b2*x[t]

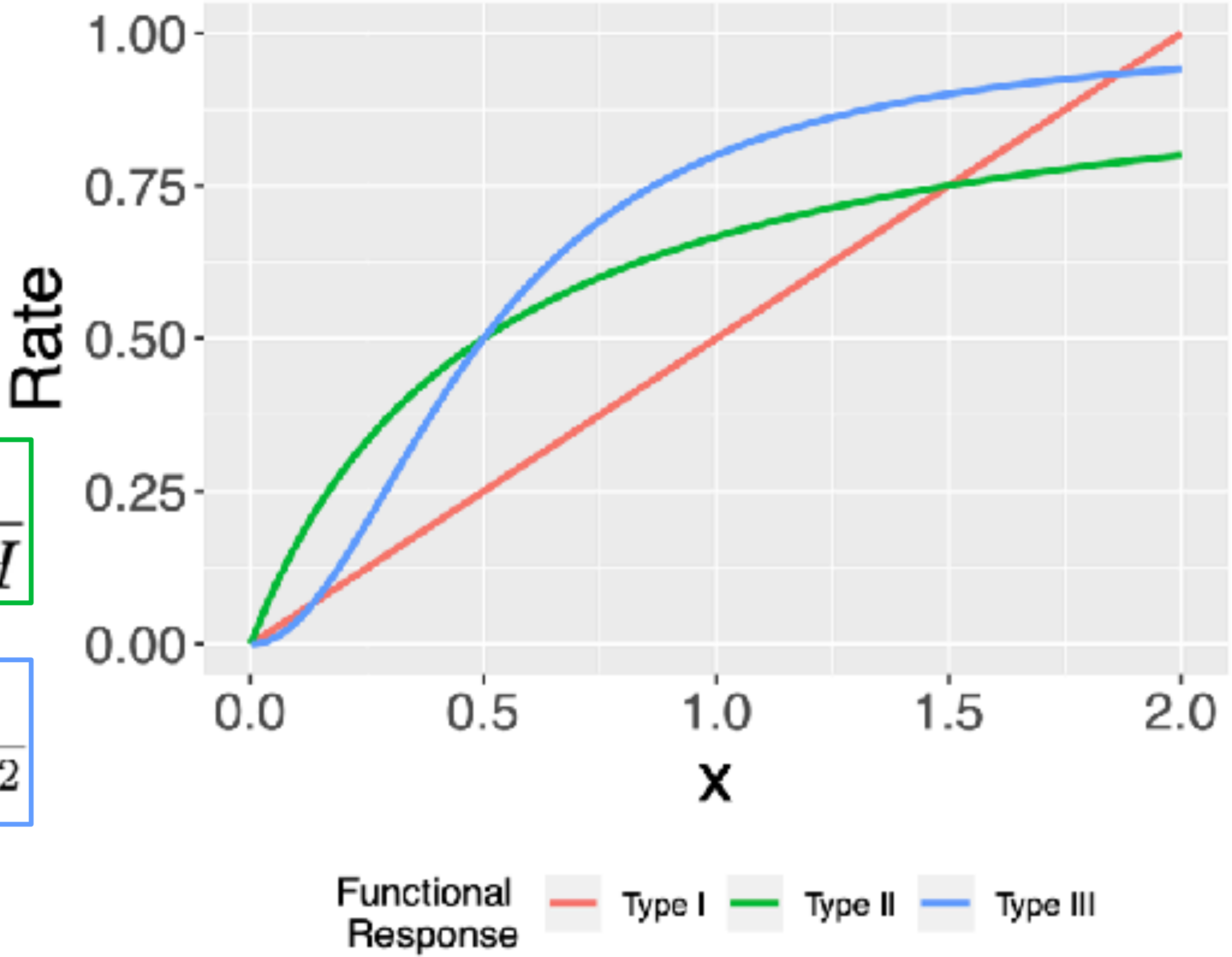


Stable trajectory
to equilibrium
Damped Oscillations
Stable
Oscillations
Chaos
Extinction
x
a
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
3.4
3.6
3.8
4

Functional Responses

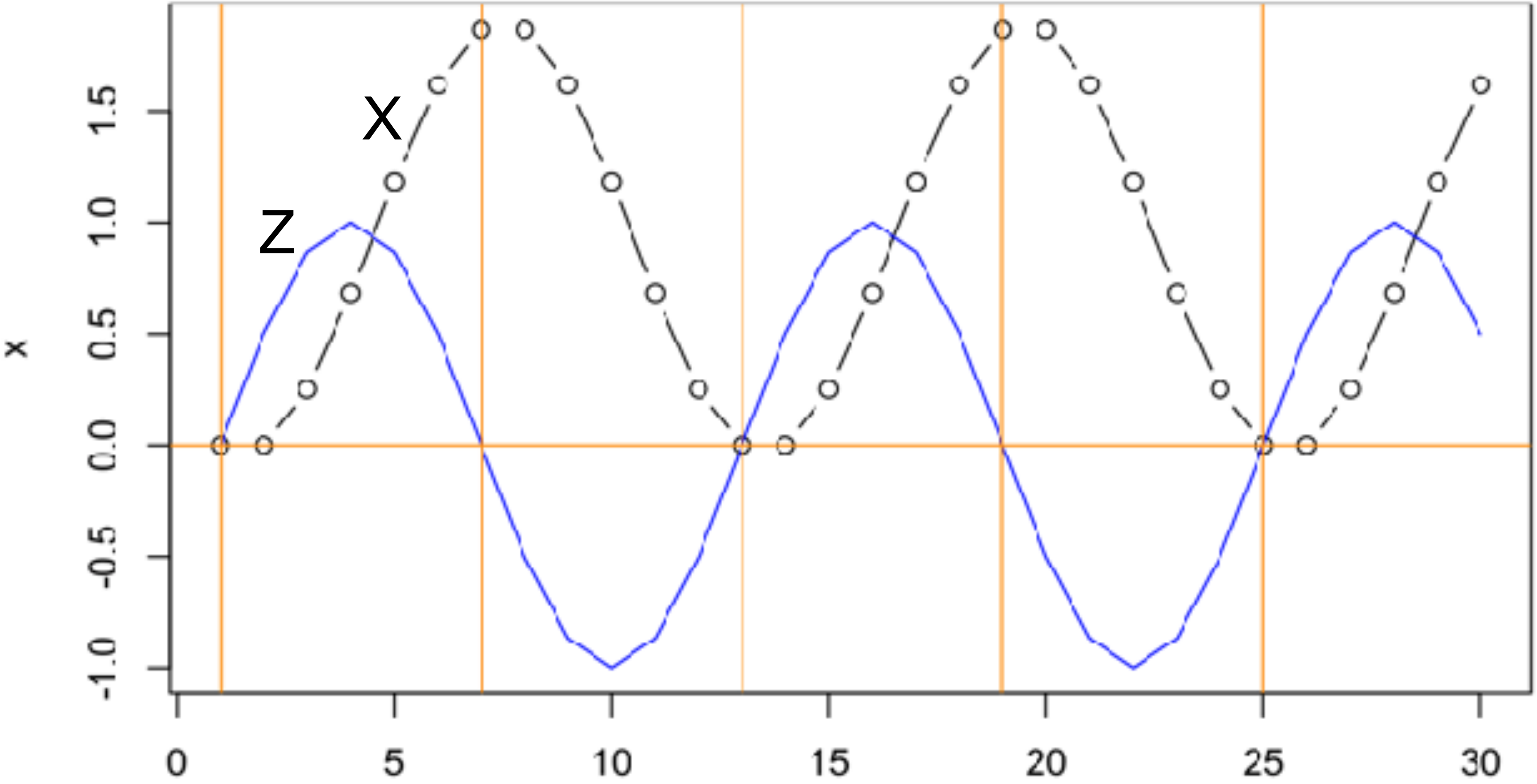
$$\frac{dH}{dt} = \frac{rH}{1 + arH}$$

$$\frac{dH}{dt} = \frac{rH^2}{1 + arH^2}$$



Zobitz 2019 Ch 3

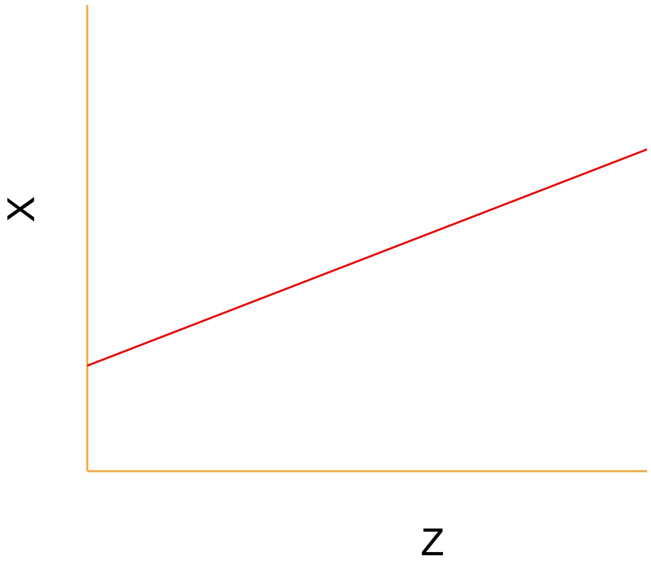
x[t+1] = x[t] + b1*Z[t]   #add a covariate

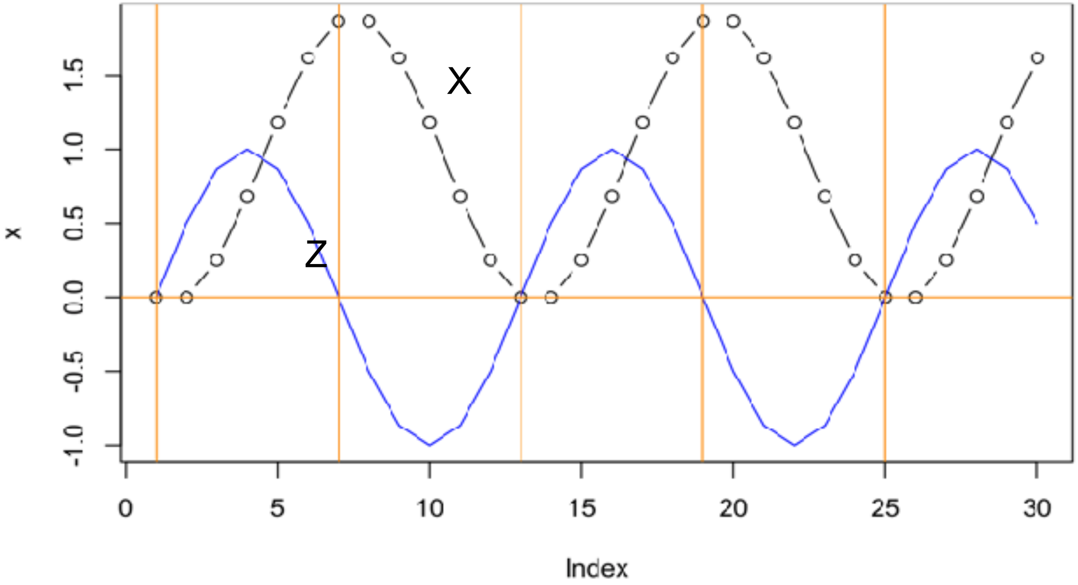


Recommendations

- Frequently useful to model covariates as **ANOMALIES** around some mean or reference point
  - Z = 0 is when covariate has no impact relative to other terms
  - $b0 + rho*X + b1*Z_{absolute}$
    - b0 is growth at Z = 0
    - strong covariance between b0 and b1
  - $b0 + rho*X + b1*Z_{anomaly}$
    - b0 is growth under 'normal' conditions
    - b1 is slope of env sensitivity
- OK to transform Z's if + and - impacts are not symmetric (e.g. drought has a stronger negative effect than pluvial has a positive effect)

Regression vs. dynamic model revisited





General dynamic model

parameter variability

$$Y_{t+1} = f(Y_t, X_t | \bar{\theta} + \alpha) + \varepsilon$$

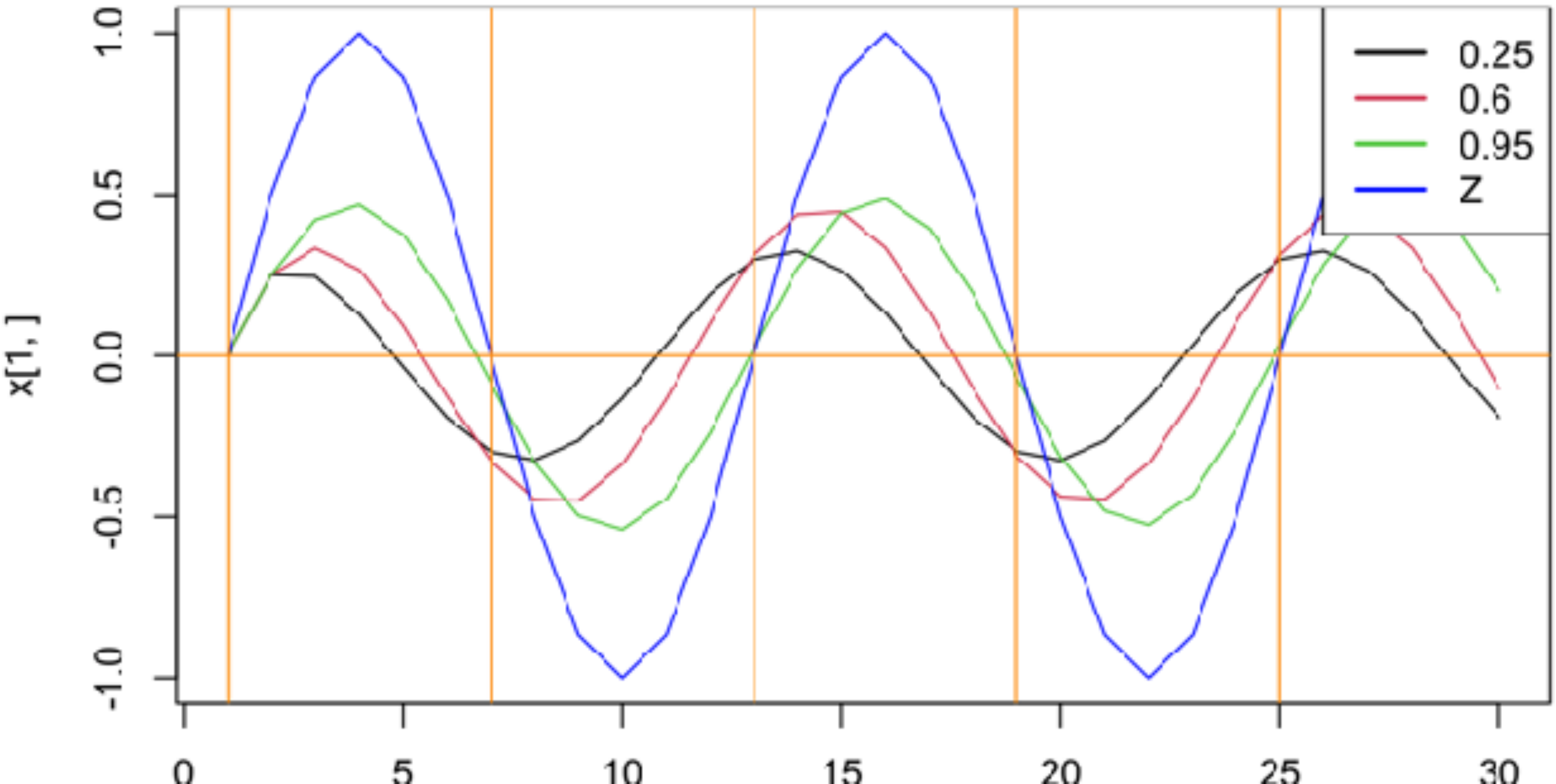
Covariates

parameters

process error

x[t+1] = rho*x[t] + b1*(Z[t+1]-Z[t])

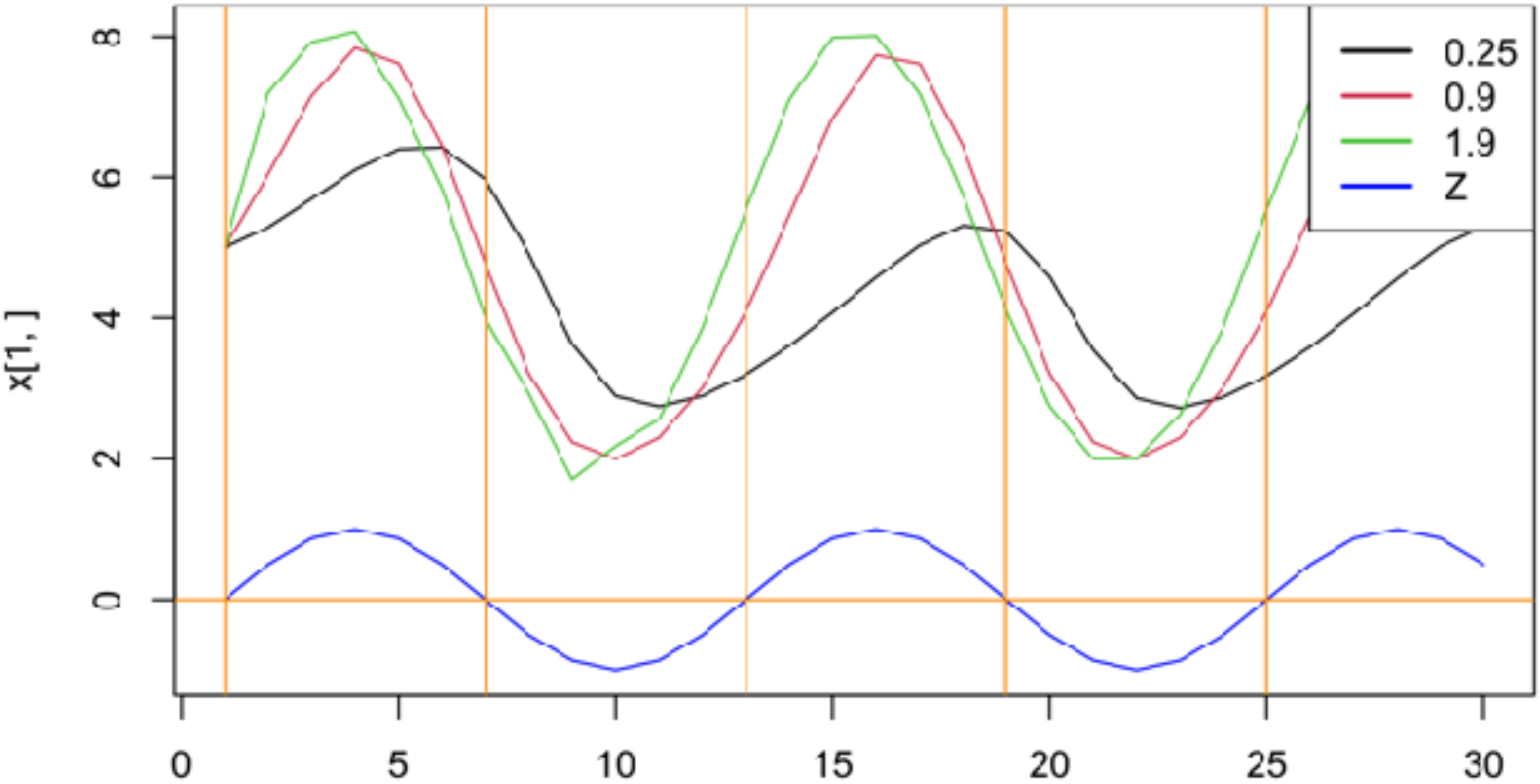
Varying rho



x[t+1] = x[t] + r*x[t]*(1-x[t]/K[t])

Logistic w/ K[t] = a0 + a1*Z[t+1]

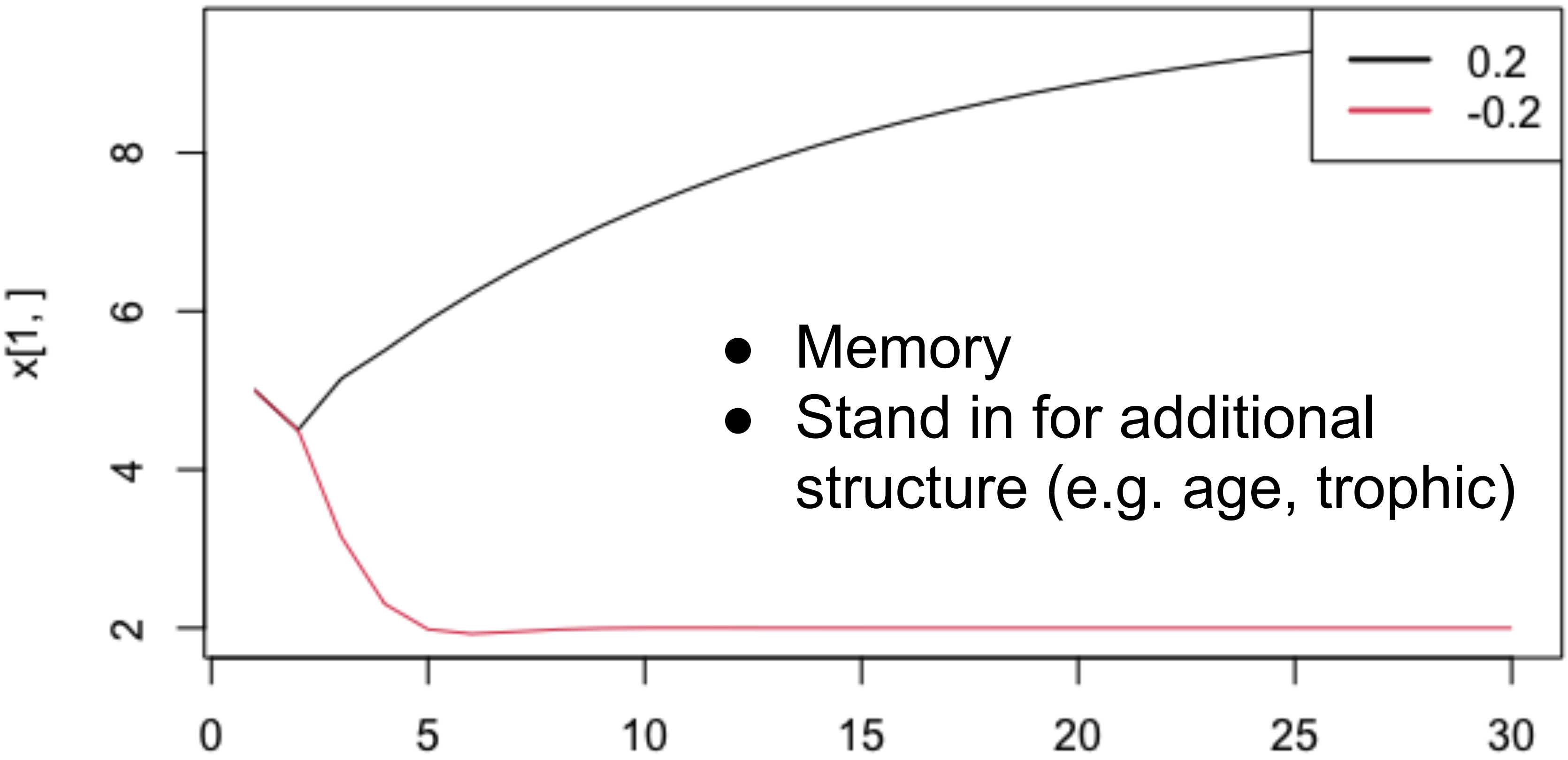
Varying r



AR(2) autoregessive

$$x[t+1] = b0 + rho*x[t] + rho2*x[t-1]$$

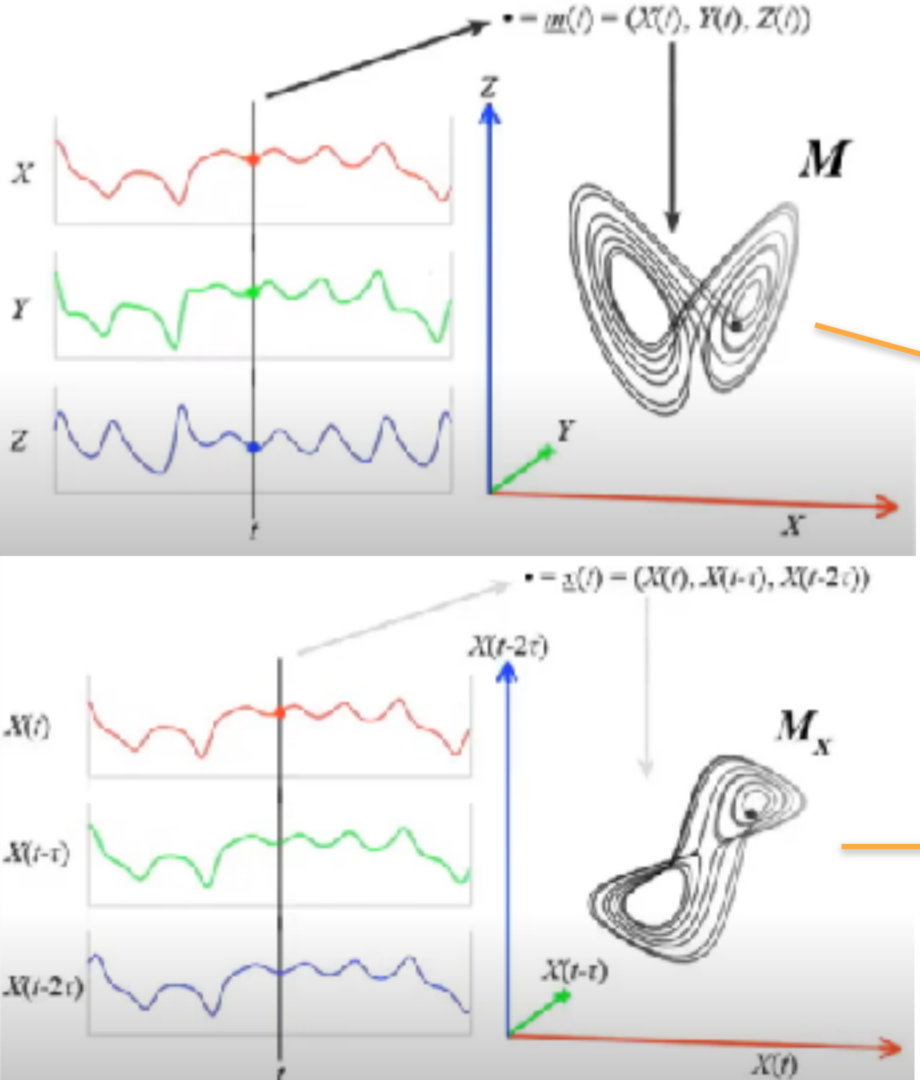
Varying rho2

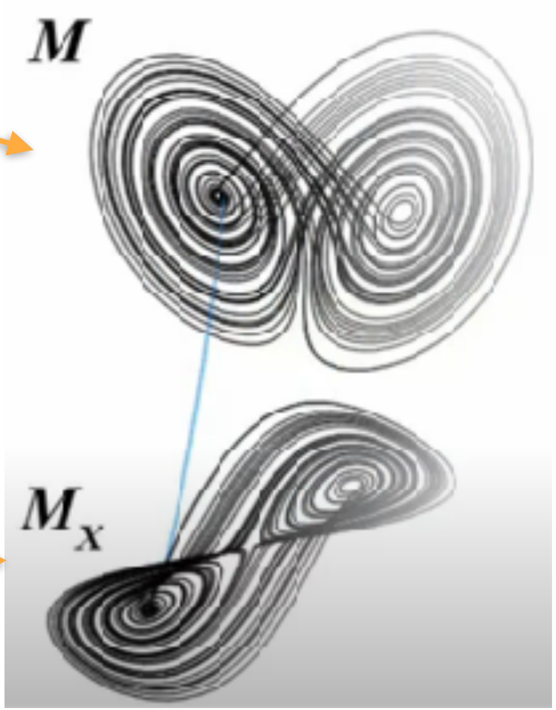


- Memory
- Stand in for additional structure (e.g. age, trophic)

Takens's theorem

Complex interactions between variables can be reconstructed from lagged versions of one variable

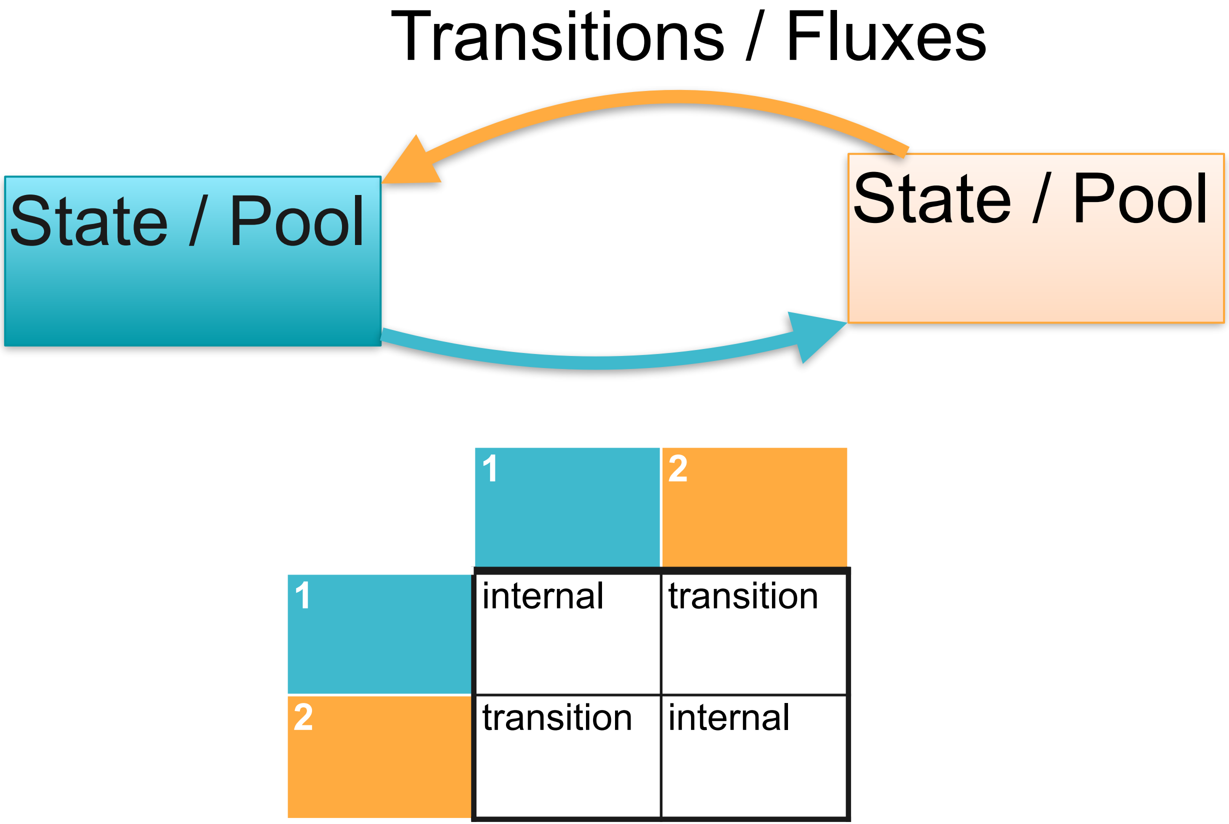




https://youtu.be/6i57udsPKms

Building Multivariate models

- Box & Arrow models
- States
  - interacting species
  - biogeochemical pools
  - age/stage classes
  - spatial locations
- Can be converted to matrices



General dynamic model

Inputs outputs

$$Y_{t+1} = Y_t + G - \mu Y_t$$

Growth

Mortality
(u proportion of Y die)

General dynamic model

Inputs outputs

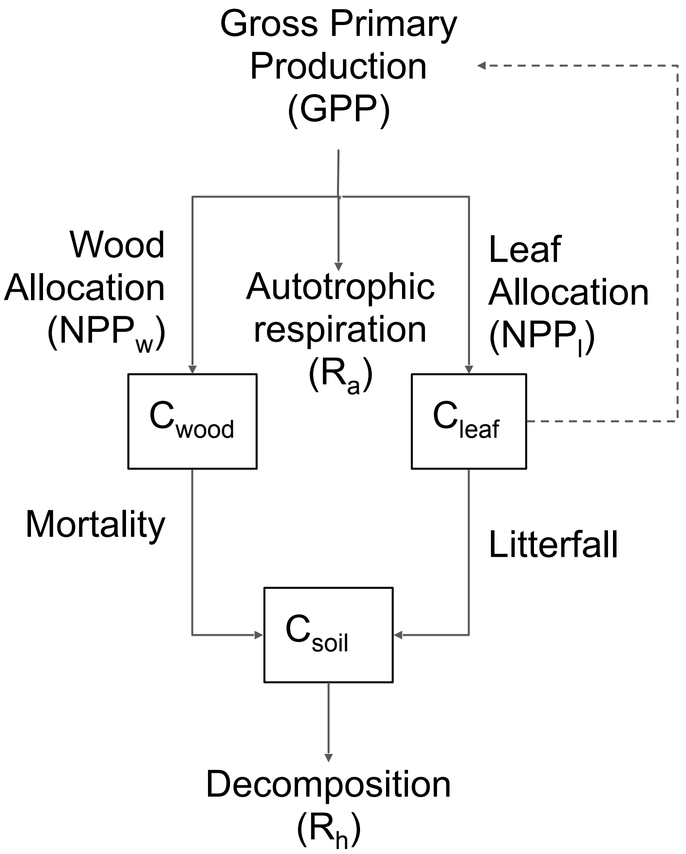
$$Y_{t+1} = Y_t + G - \mu Y_t$$

Growth

Mortality
(u proportion of Y die)

$$G = f(\text{factors that influence growth})$$

A simple ecosystem process model



Develop balance equations

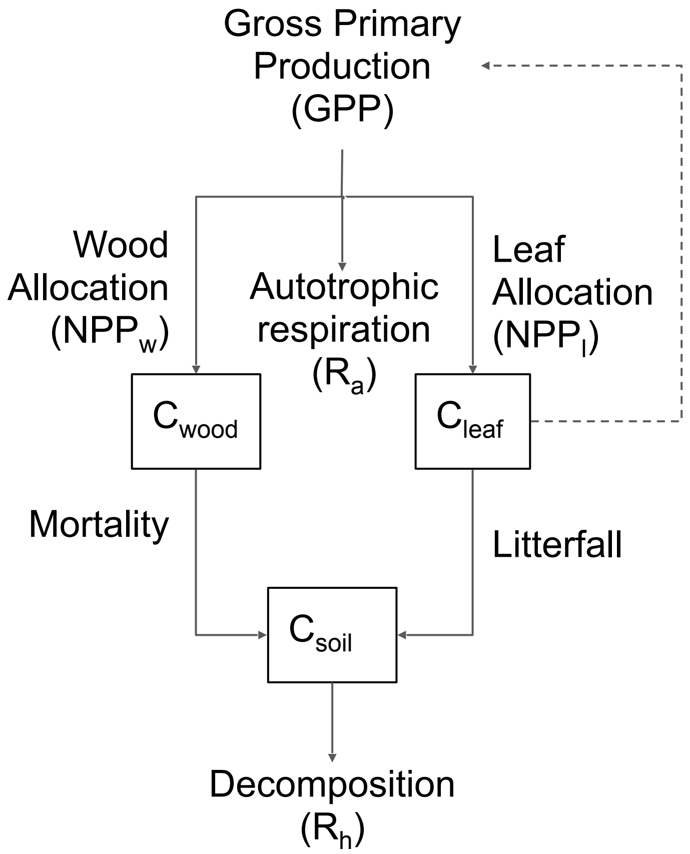
$C_{leaf}[t + 1] = C_{leaf}[t] + \mathbf{NPP_L}$ - litterfall

$C_{wood}[t + 1] = C_{wood}[t] + NPP_W$ - mortality

$C_{soil}[t + 1] = C_{soil}[t]$ + mortality + litterfall - **Rh**

Functions of Covariates (PAR & Temperature)

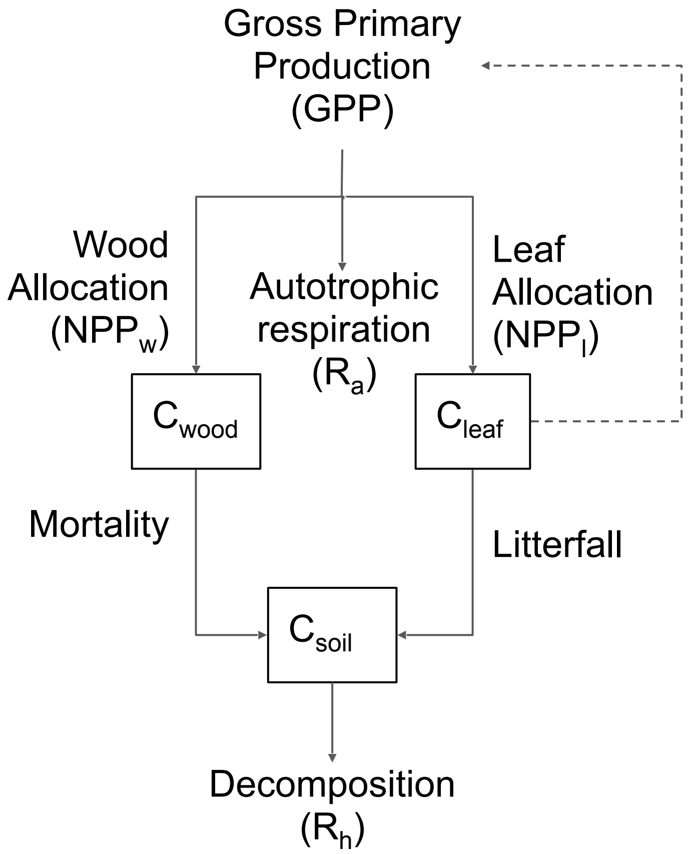
A simple ecosystem process model



Start with a conceptual model

- Boxes = states
- Solid arrows = fluxes
- Dashed arrows = influences

A simple ecosystem process model



Develop balance equations

$C_{leaf}[t + 1] = C_{leaf}[t] + \mathbf{NPP_L}$ - litterfall

$C_{wood}[t + 1] = C_{wood}[t] + NPP_W$ - mortality

$C_{soil}[t + 1] = C_{soil}[t]$ + mortality + litterfall - **Rh**

Functions of Covariates (PAR & Temperature)

Develop flux equations

$NPP_L = GPP[t] * \mathit{falloc}_L$

GPP[t] = *alpha* * **PAR[t]** * (1- exp(-0.5 * LAI[t]))

$LAI[t] = SLA * C_{leaf}[t]$

Italics: parameters that need to be estimated

Takeaways

- Start simple, build complexity incrementally
- Dynamic models: $X_{t+1} = X_t$ + CHANGE
- State variables & transitions/fluxes (rates of change)
- Intercept = change independent of state variable (rare?)
- Can add complexity by making parameters functions of other things
- Covariates: absolute, anomaly, change (dZ)
    - dynamic models rarely predict synchrony between drivers and responses
- Functional responses (both endogenous & exogenous) need not be linear
- Takens's Theorem (lags can be useful)
- Boxs and Arrows
    - matrix models are ubiquitous in ecology