State-Space Models

State-Space Models….

- future state depends on current state (dynamic model)

$$X_{t+1} \sim X_t$$

State-Space Models….

- future state depends on current state (dynamic model)

$$X_{t+1} \sim X_t$$

- Observe variable Y but want to interpret response X
  [when specific to time – Hidden **Markov** Model]

State-Space Models….

For when you observe this state (at time t) :

Y_t

But want to estimate this:

X_t

State-Space Model

$$X_t = f(X_{t-1}) + \varepsilon_t$$ **Process Model**

$$Y_t = g(X_t) + \omega_t$$ **Data Model**

- X = latent time series
- Y = observed data
- ε = process error
- ω = observation error

Random Walk State-Space Model

$$X_t = X_{t-1} + \varepsilon_t$$ **Process Model**

$$Y_t = g(X_t) + \omega_t$$ **Data Model**

- X = latent time series
- Y = observed data
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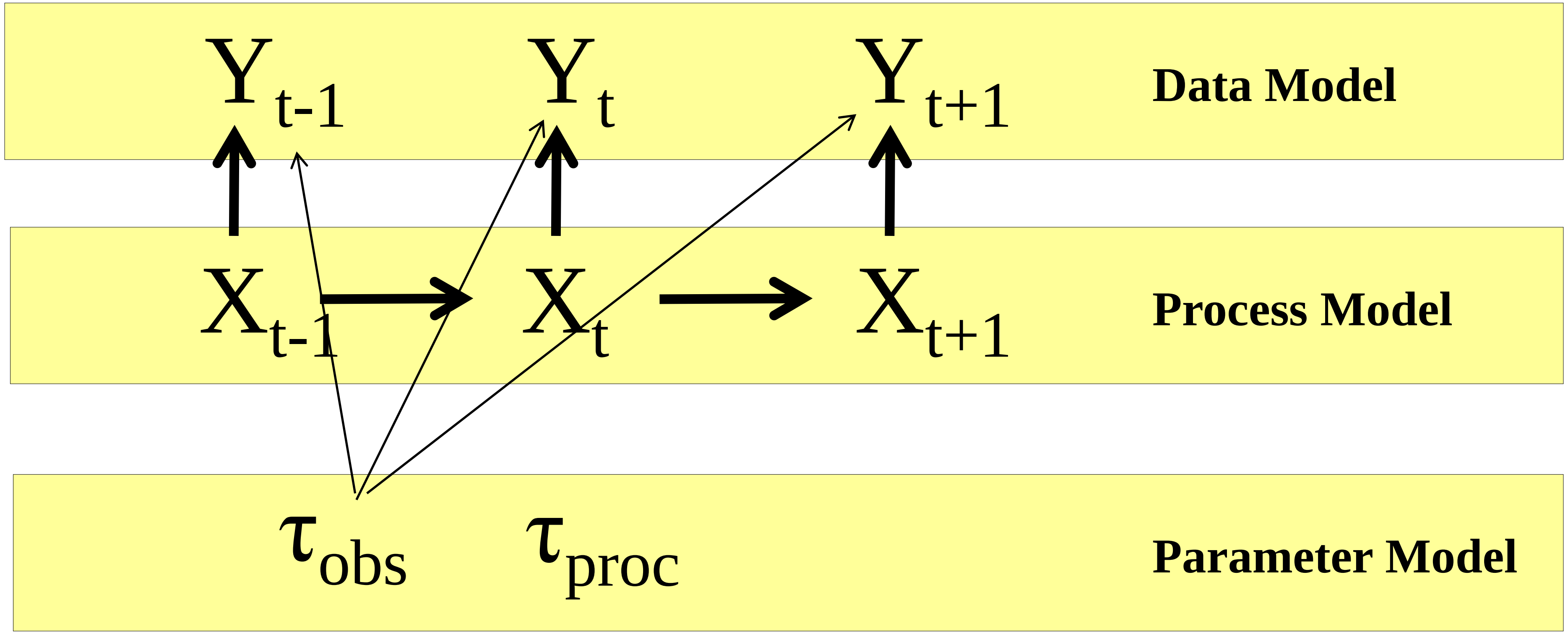
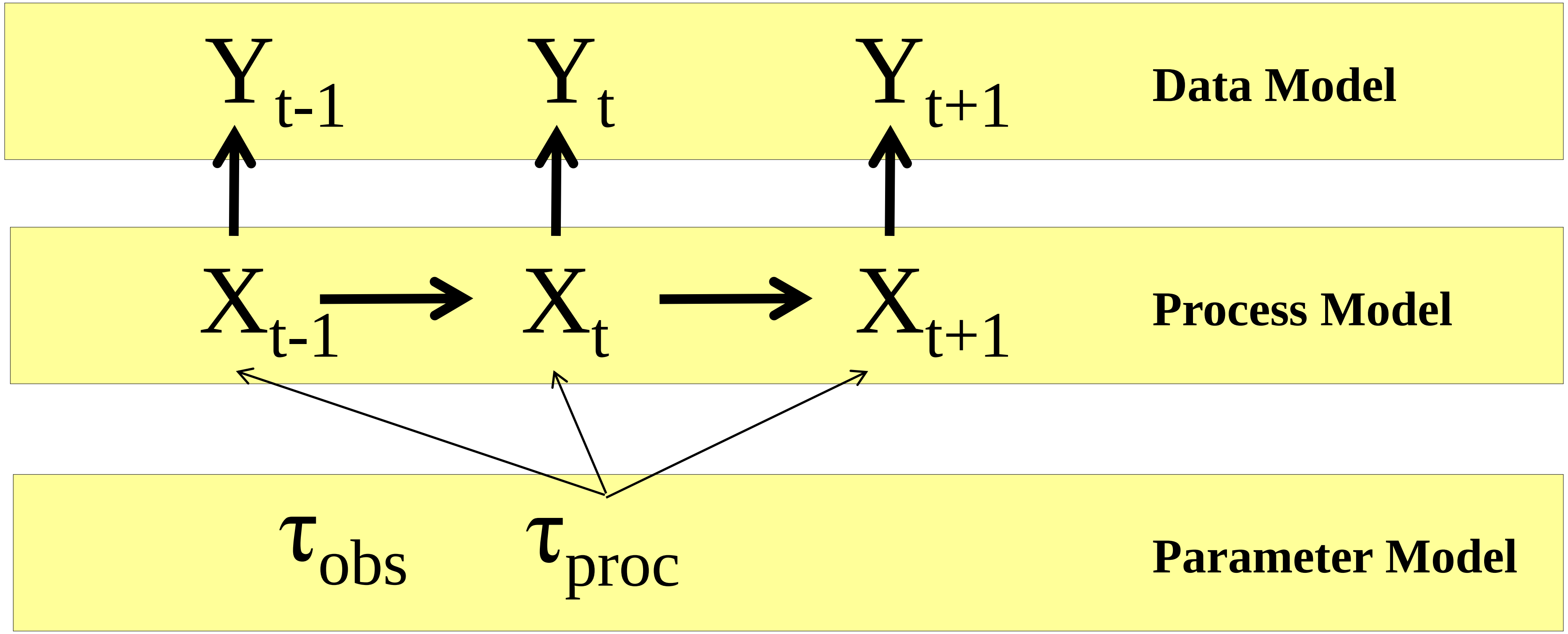
Random Walk State-Space Model



Y's are conditionally independent given the X's

Random Walk State-Space Model



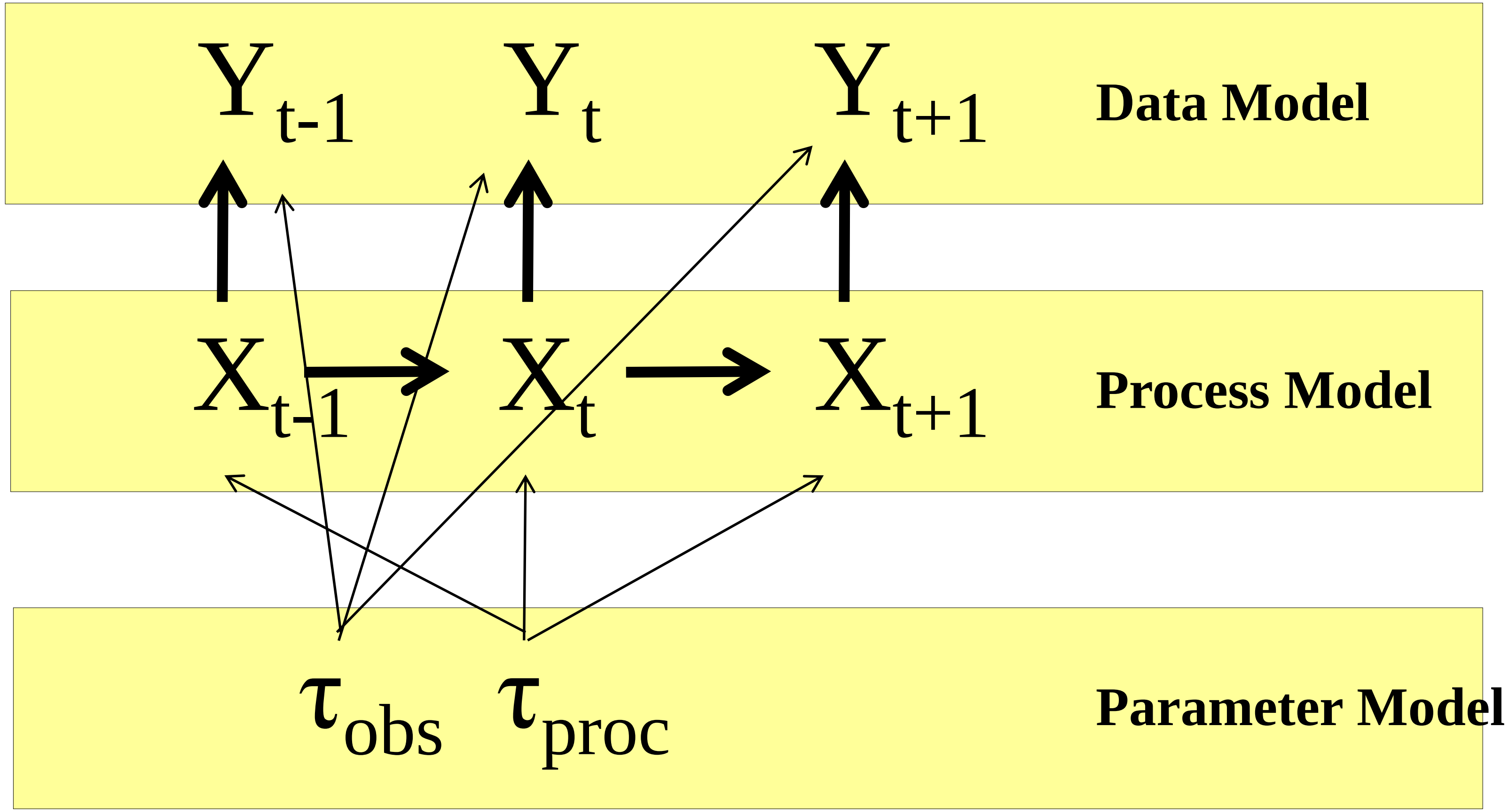
X_t estimate depends on X_{t+1}, X_{t-1} and Y_t

```
RandomWalk = "
model{

  #### Data Model
  for(i in 1:n){
    y[i] ~ dnorm(x[i],tau_obs)
  }

  #### Process Model
  for(i in 2:n){
    x[i]~dnorm(x[i-1],tau_proc)
  }

  #### Priors
  x[1] ~ dnorm(x_ic,tau_ic)
  tau_obs ~ dgamma(a_obs,r_obs)
  tau_proc ~ dgamma(a_proc,r_proc)
}
```



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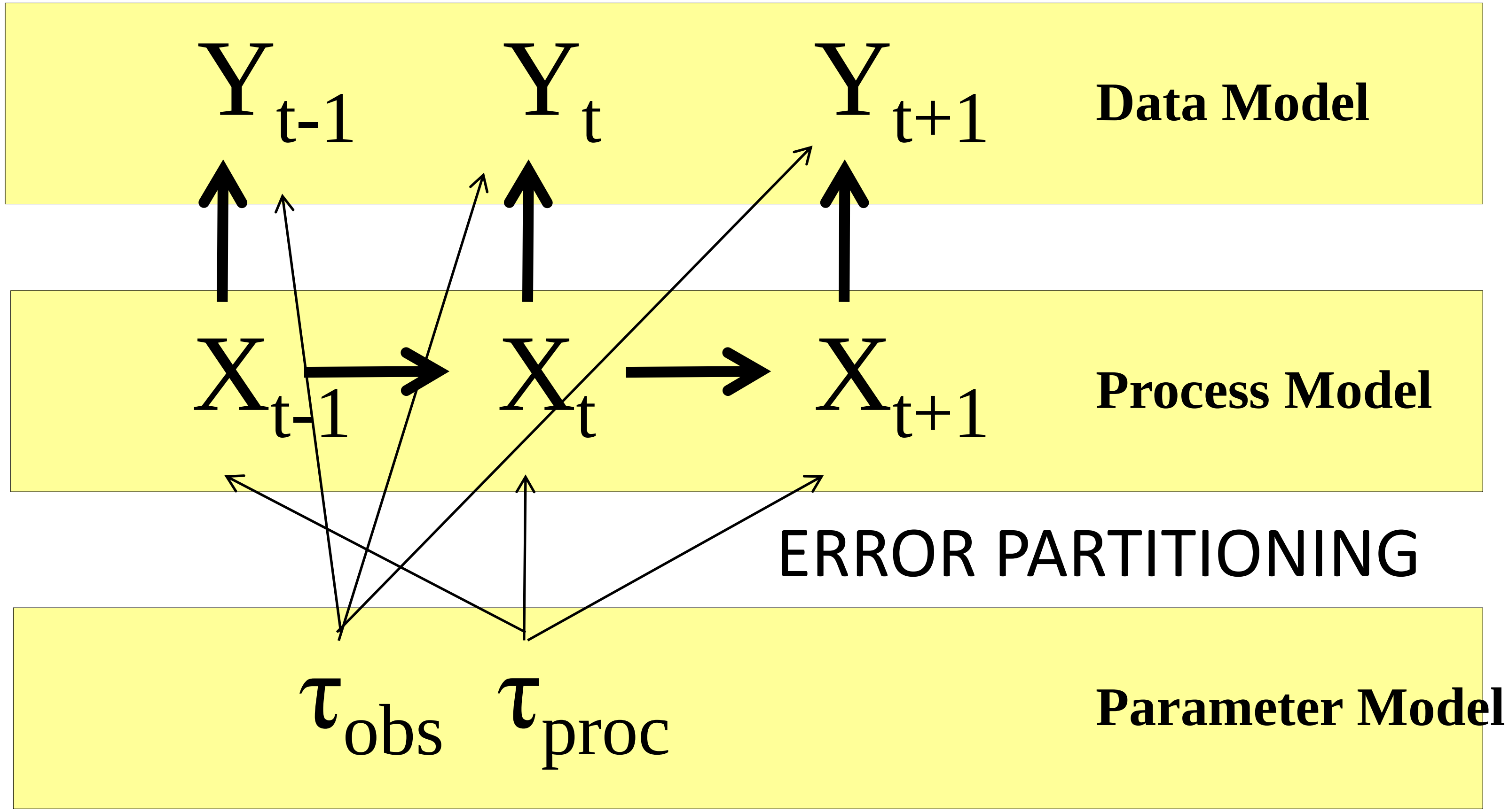
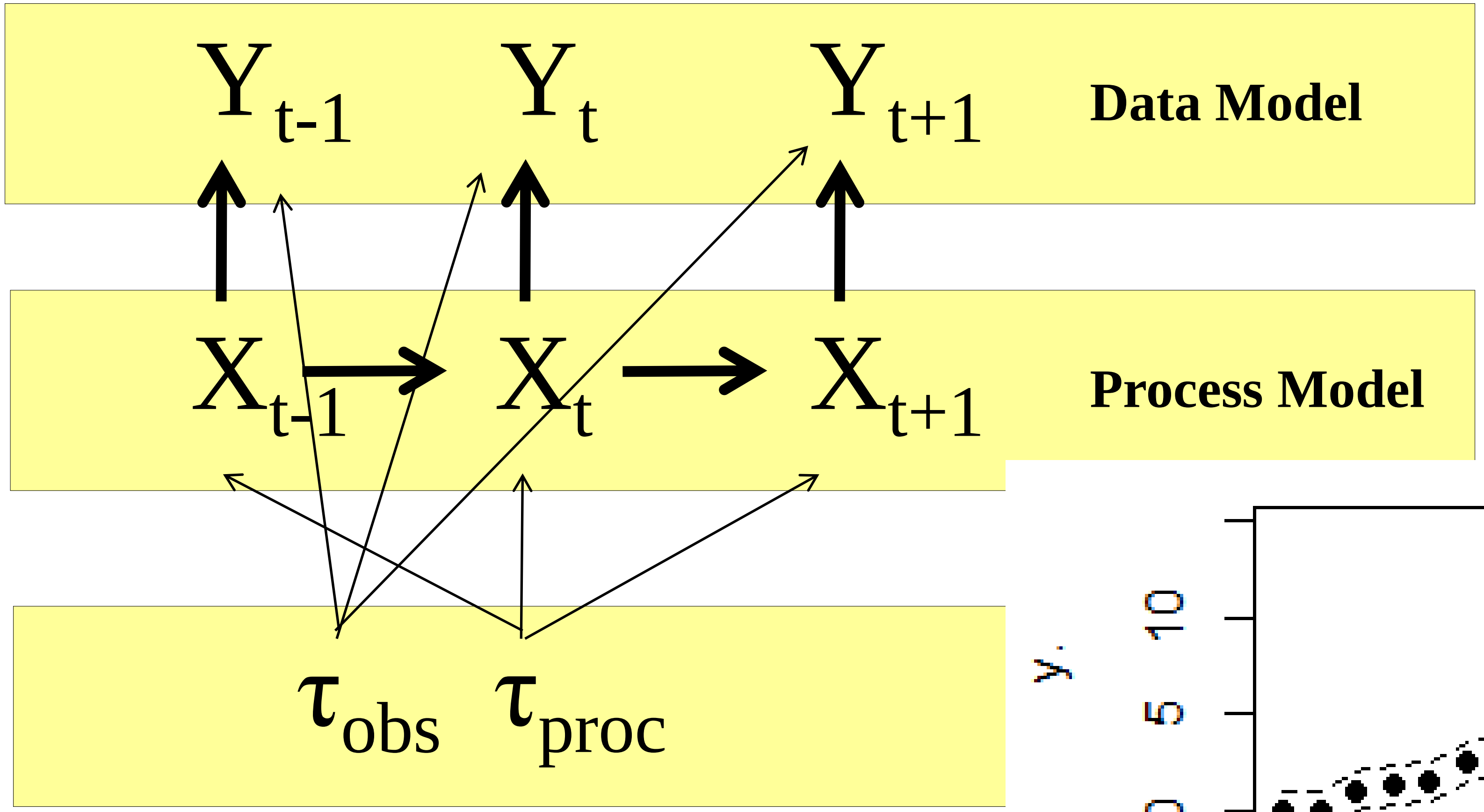
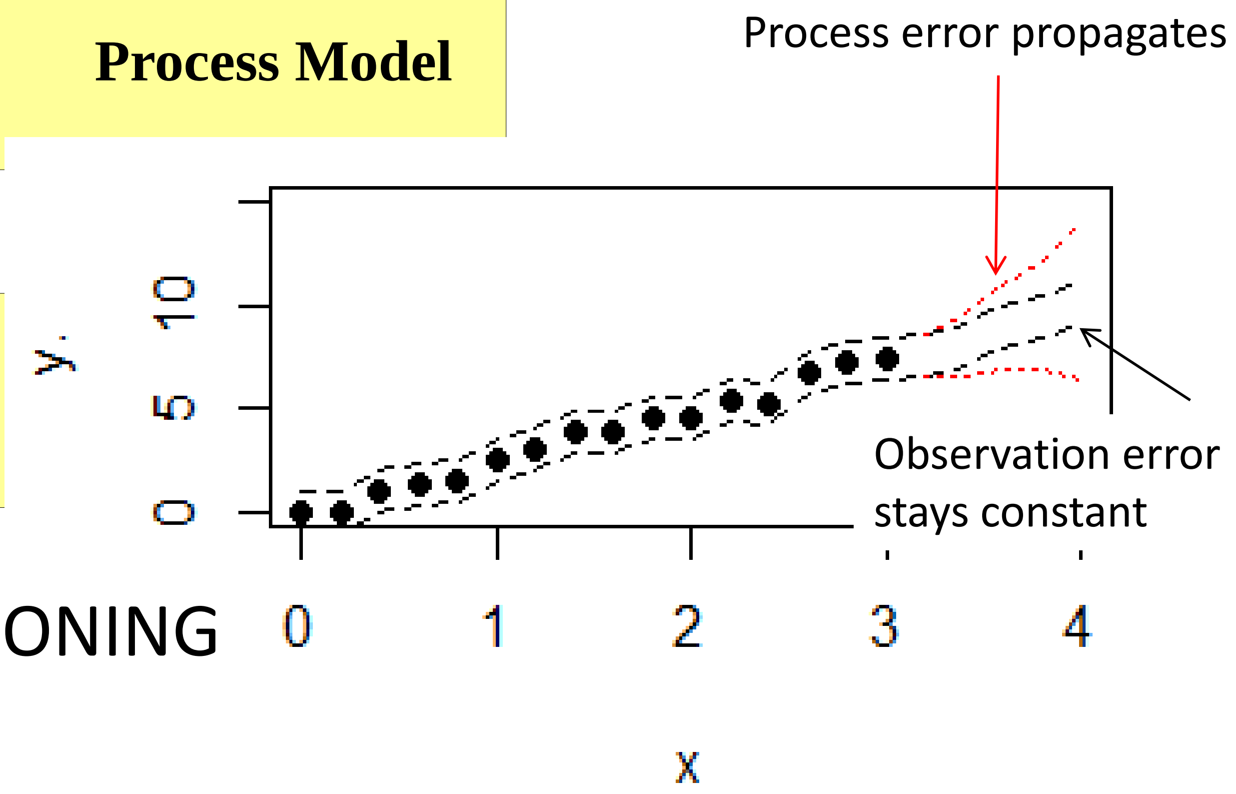


X_t depends on X_{t+1} and X_{t-1} and their uncertainties (τ_{proc})



ERROR PARTITIONING



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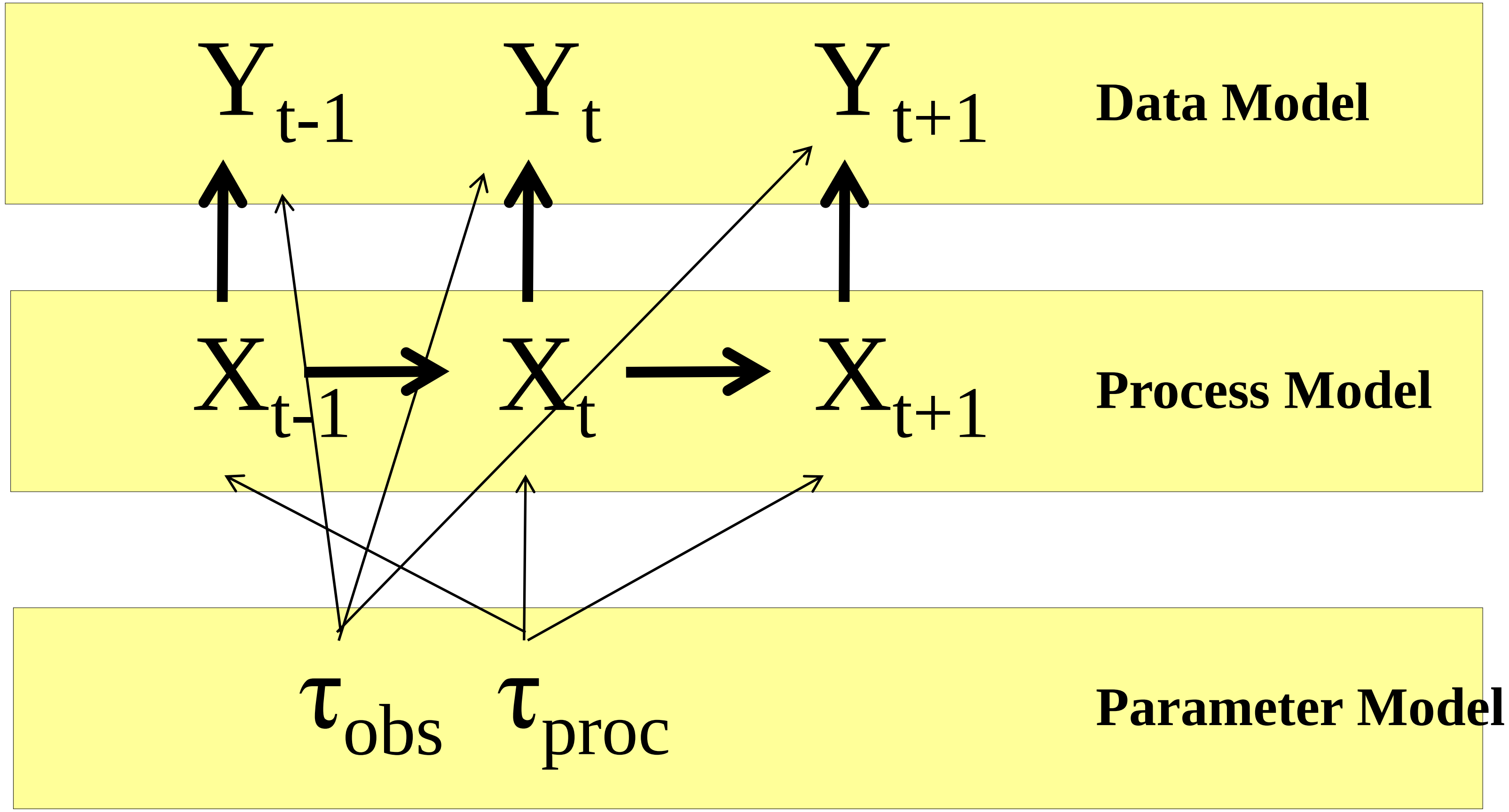
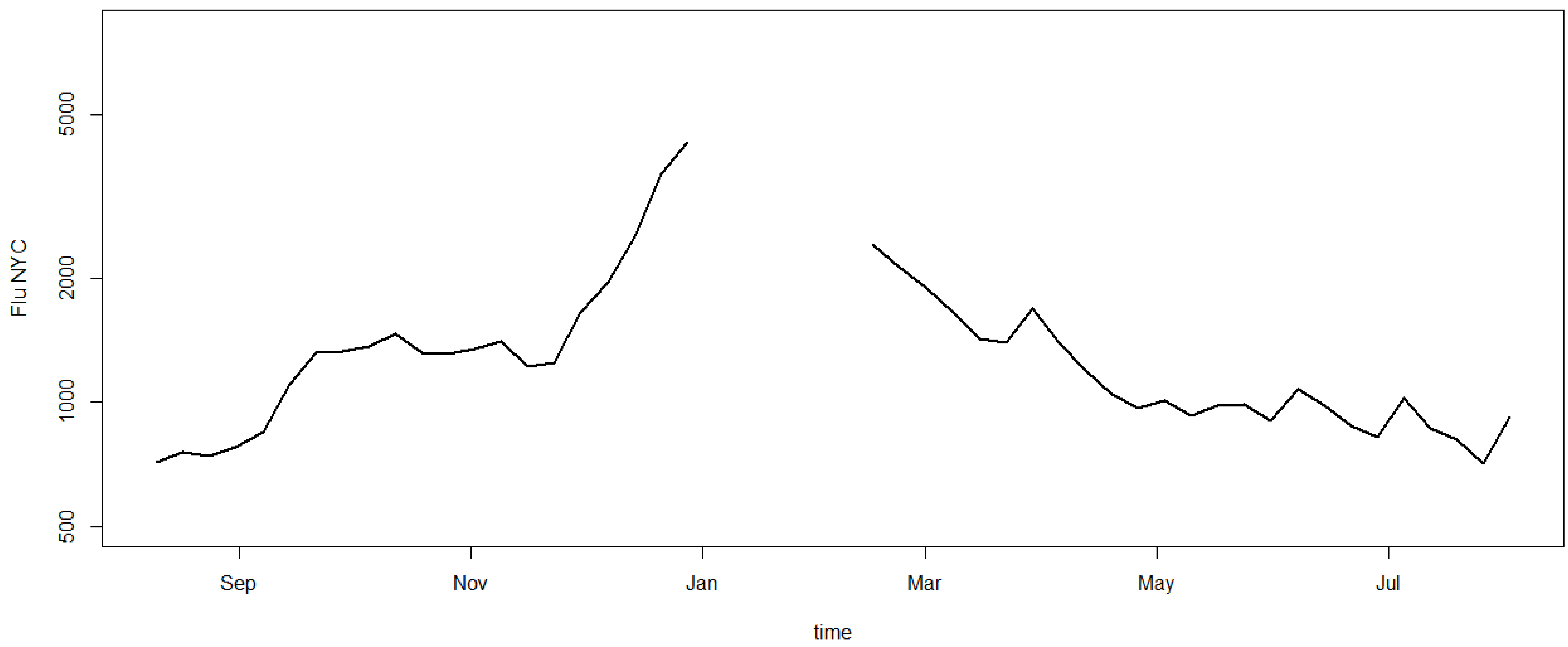
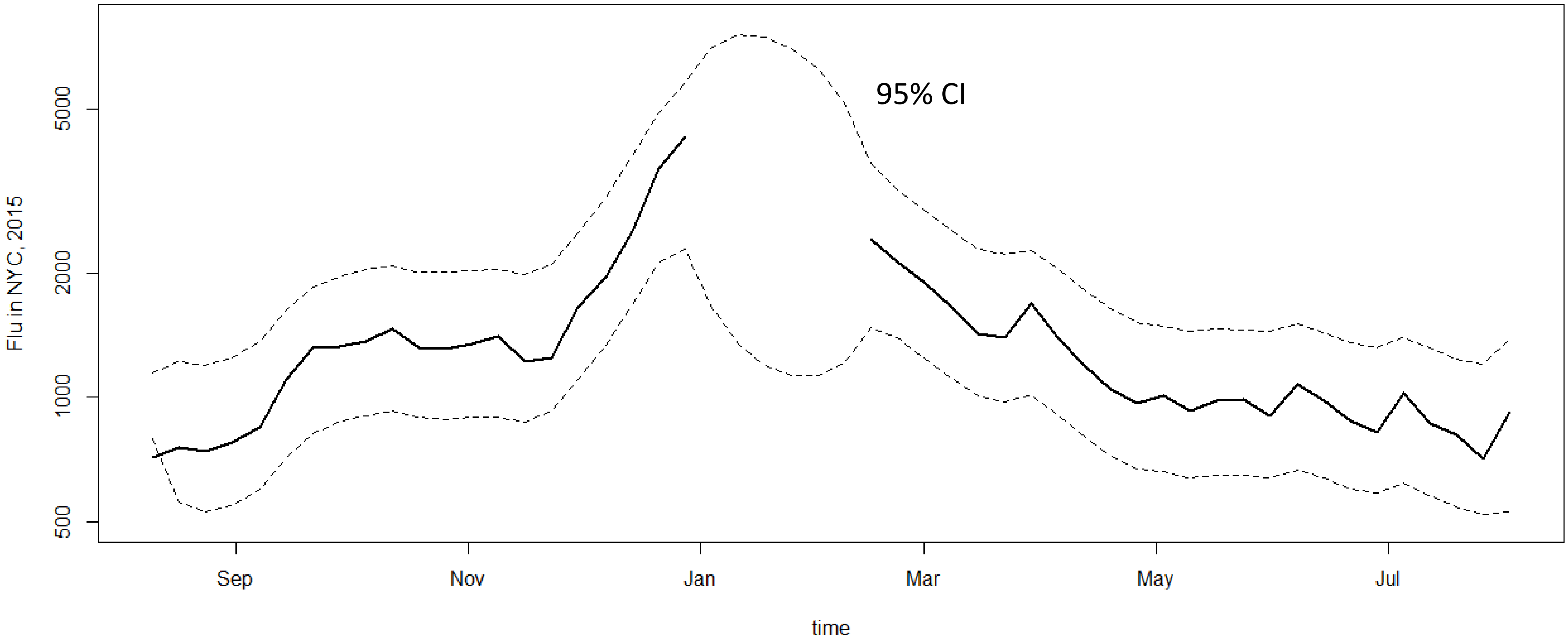
```



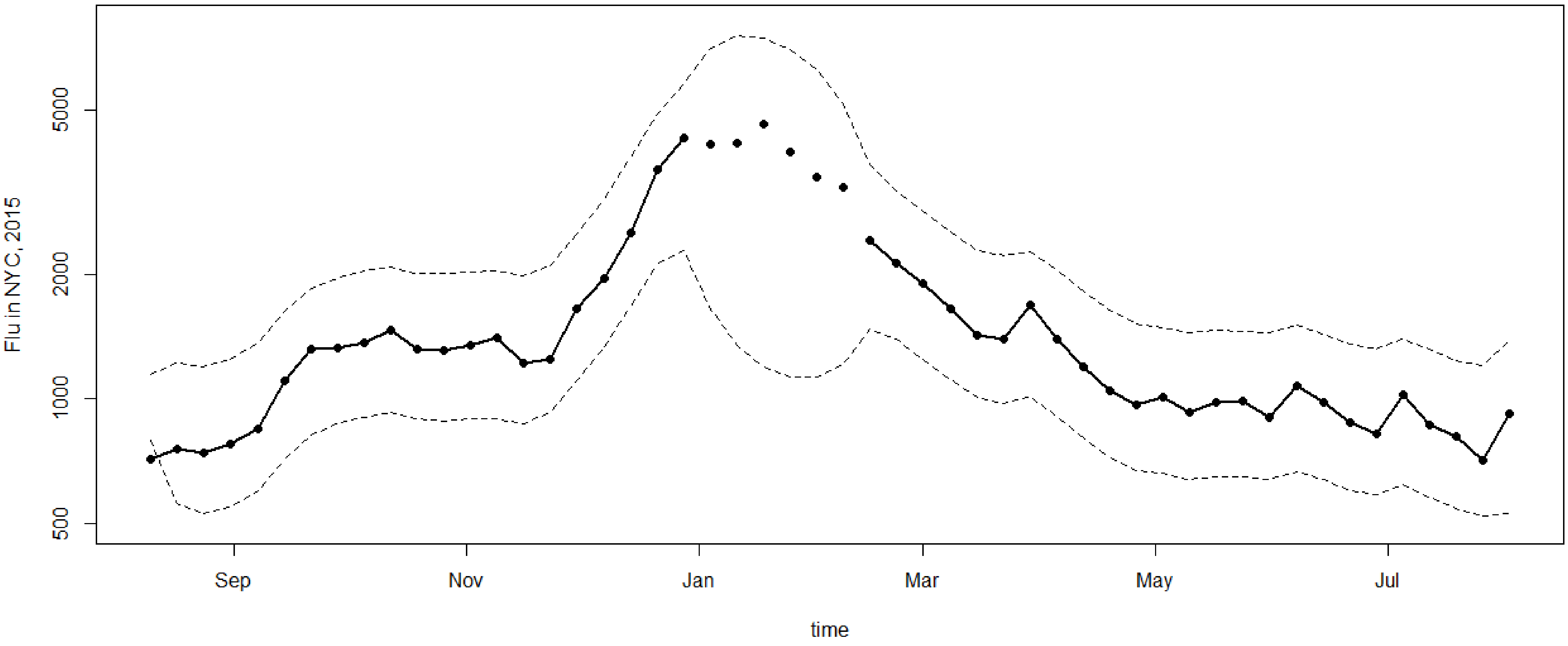
Random Walk State Space Model



Random Walk State Space Model



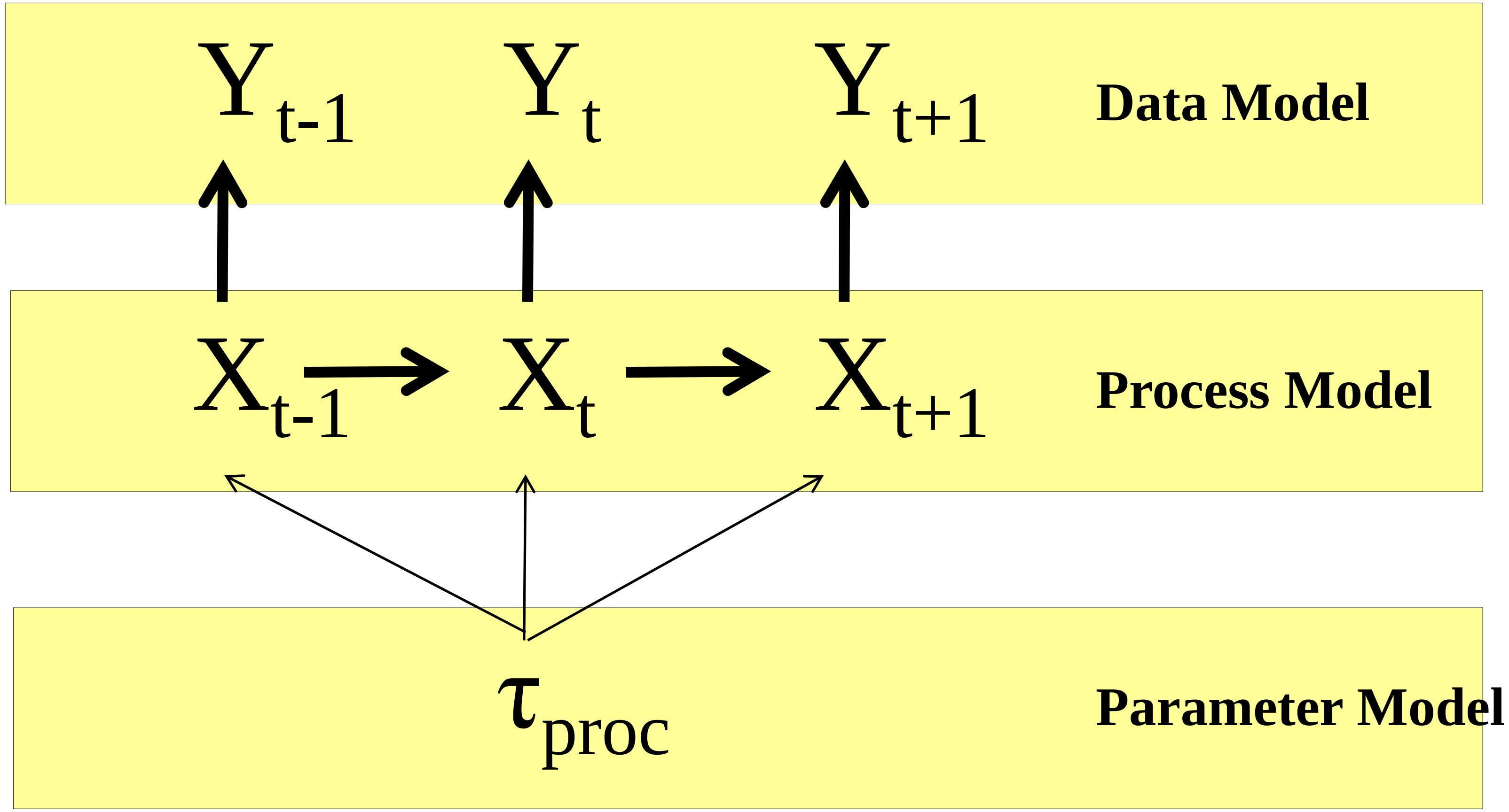
Random Walk State Space Model



Beyond the Gaussian Random Walk

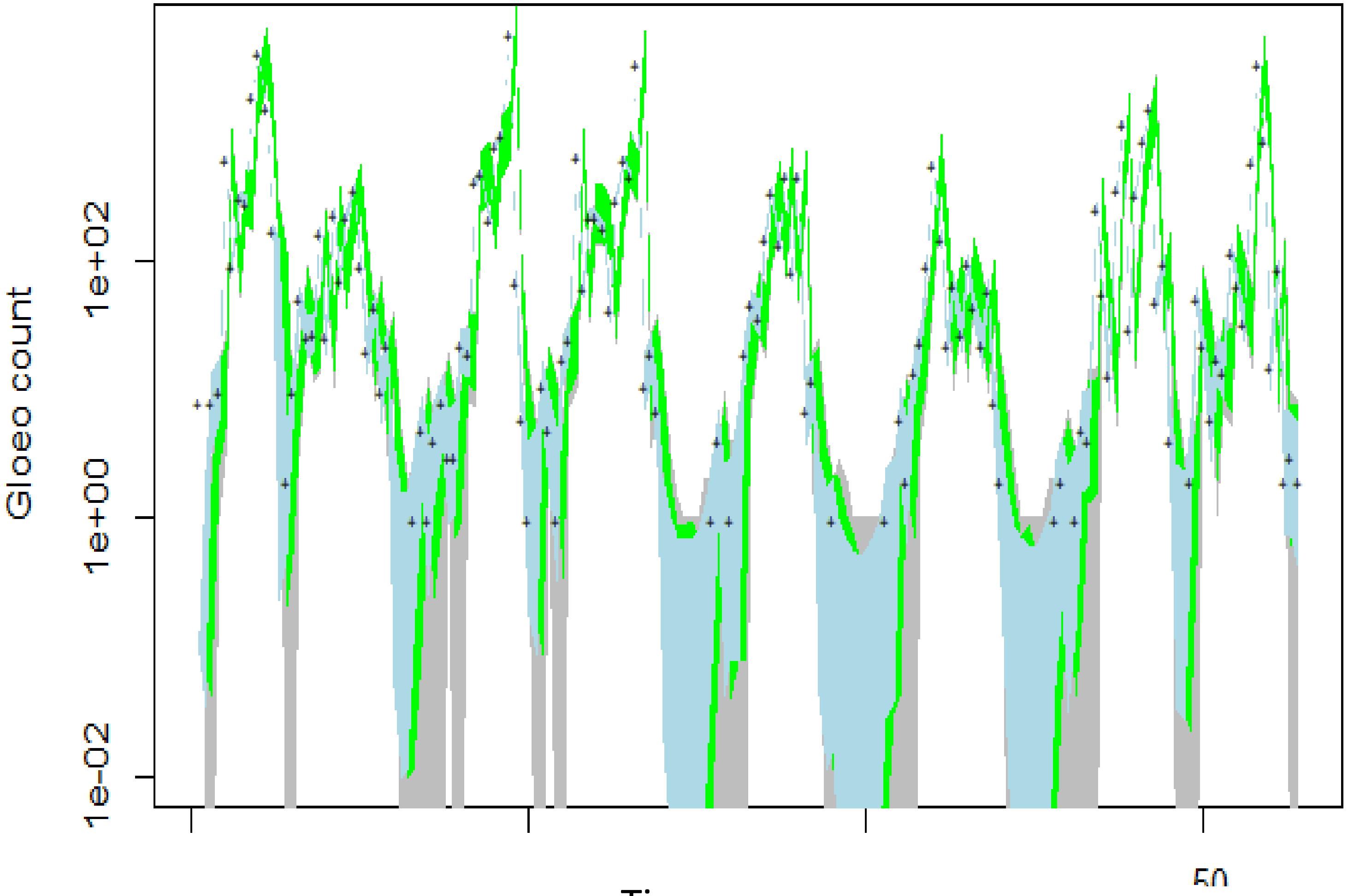
- Non-Gaussian data model
- Covariates
- Detection Models

Algal Bloom Example – Poisson RW



GLEON: Lofton, Brentrup, Beck, Zwart, Bhattacharya,McCullough

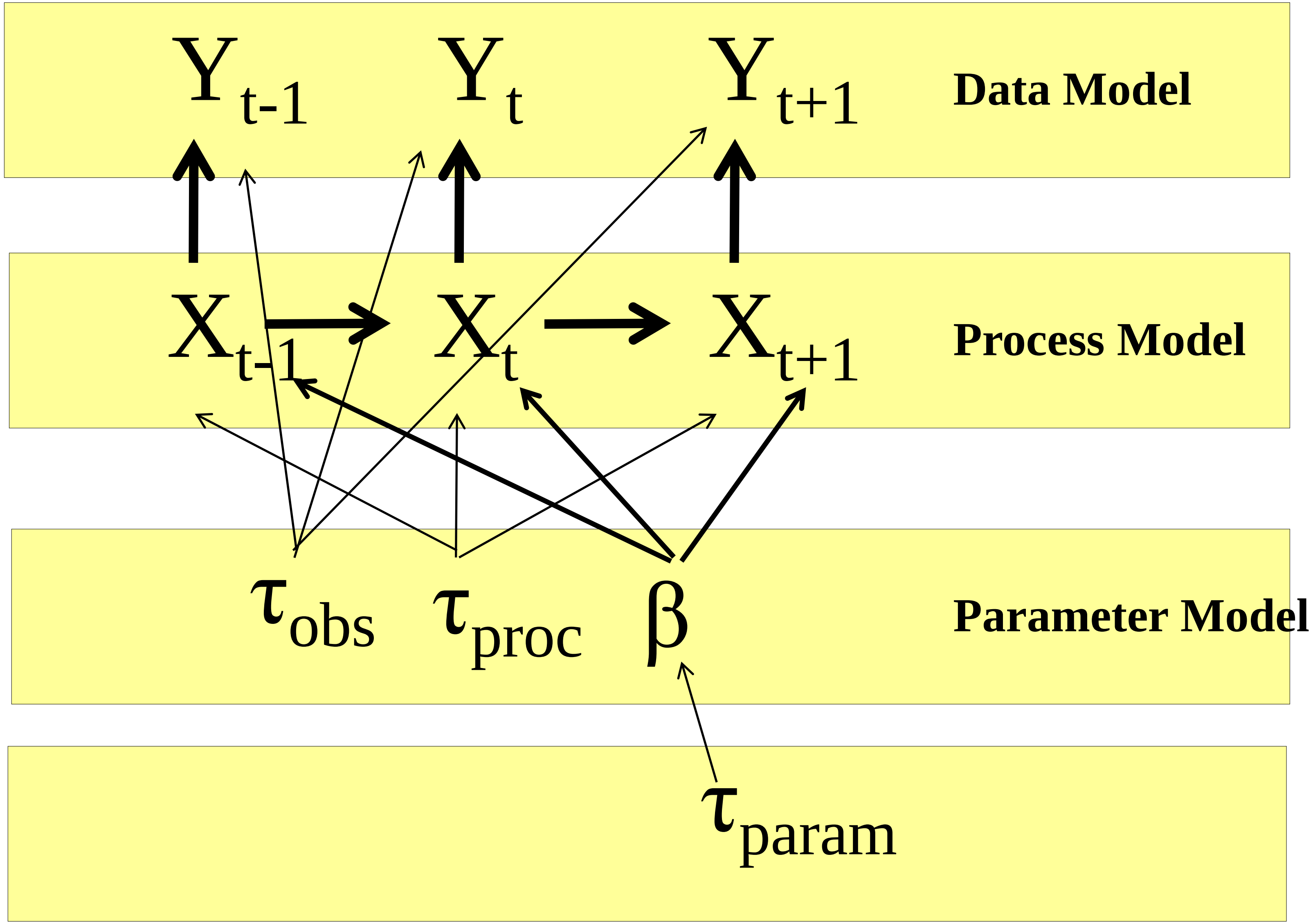
Algal Bloom Example – Poisson RW



Green = model fit
Blue = predictive interval
Gray = predictive interval + data model

GLEON: Lofton, Brentrup, Beck, Zwart, Bhattacharya,McCullough

Jake Zwart – GIF showing 1-step ahead forecast RW



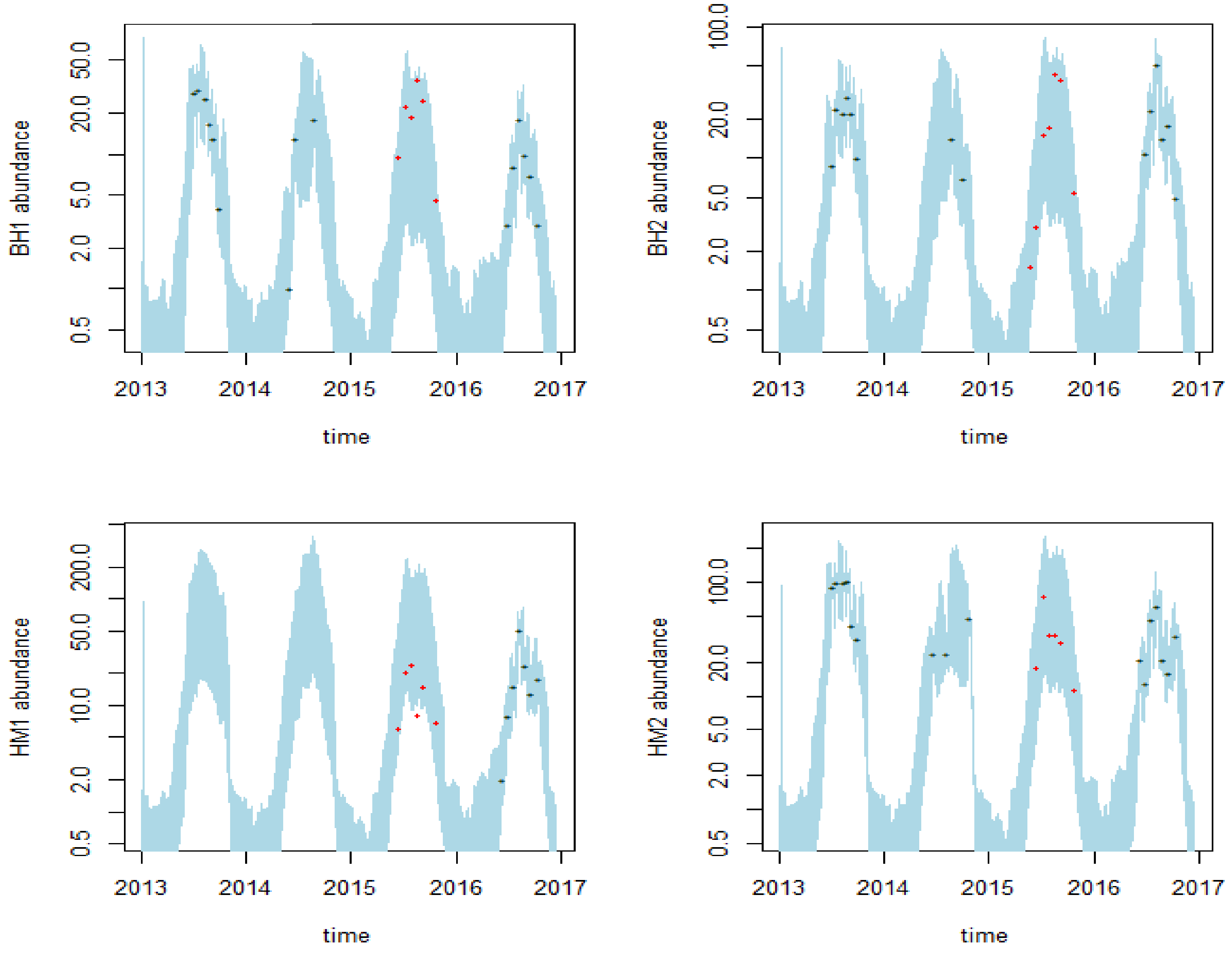
Y_{t-1}
Y_t
Y_{t+1}
Data Model
X_{t-1}
X_t
X_{t+1}
Process Model
τ_{obs}
τ_{proc}
β
Parameter Model
τ_{param}

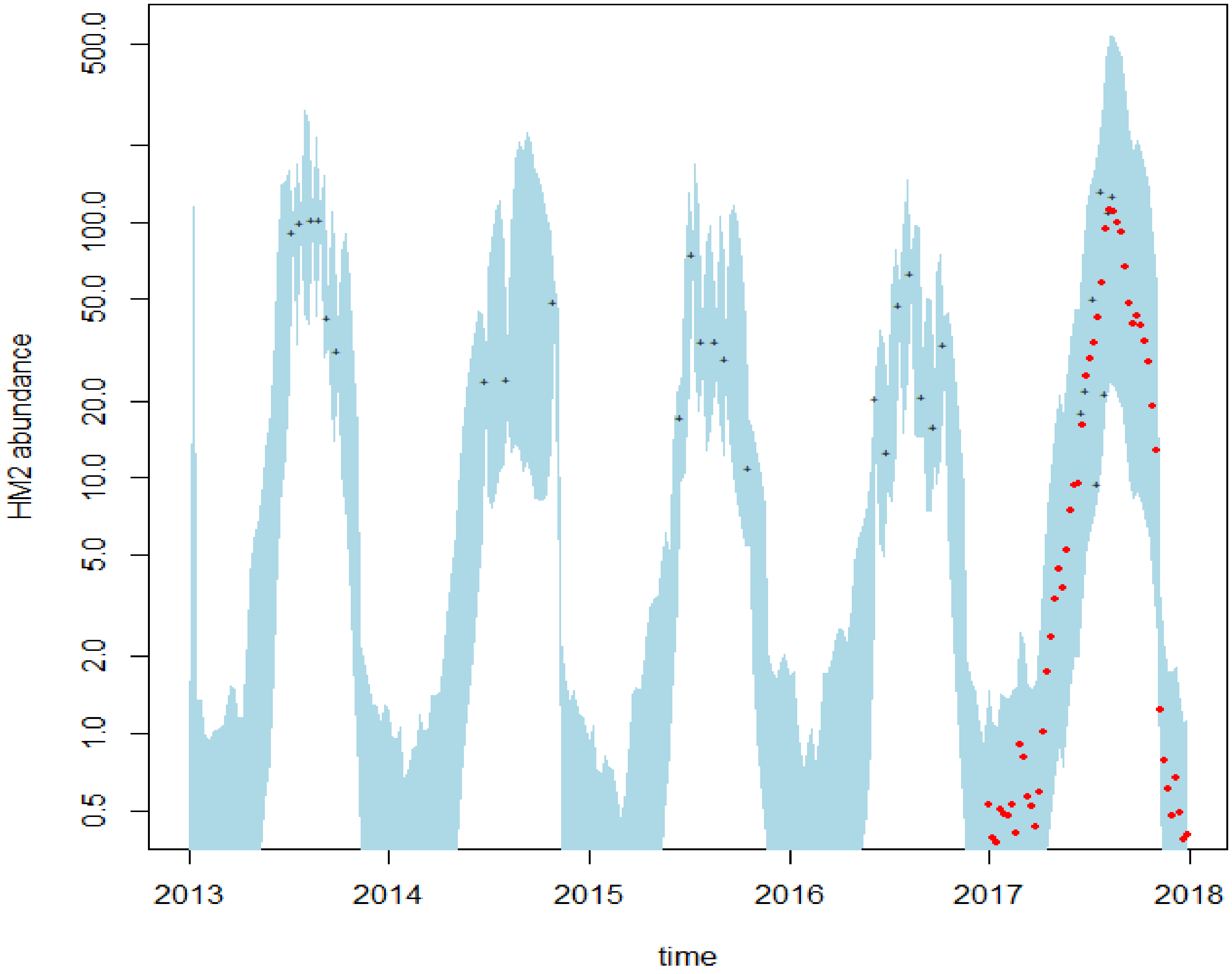
Process Model

$$X_{it} = f\left(X_{i,t-1} + weathert + juv\ habitati\right) + \varepsilon_{it}$$

Data Model

$$Y_{it} = g(X_{it})$$



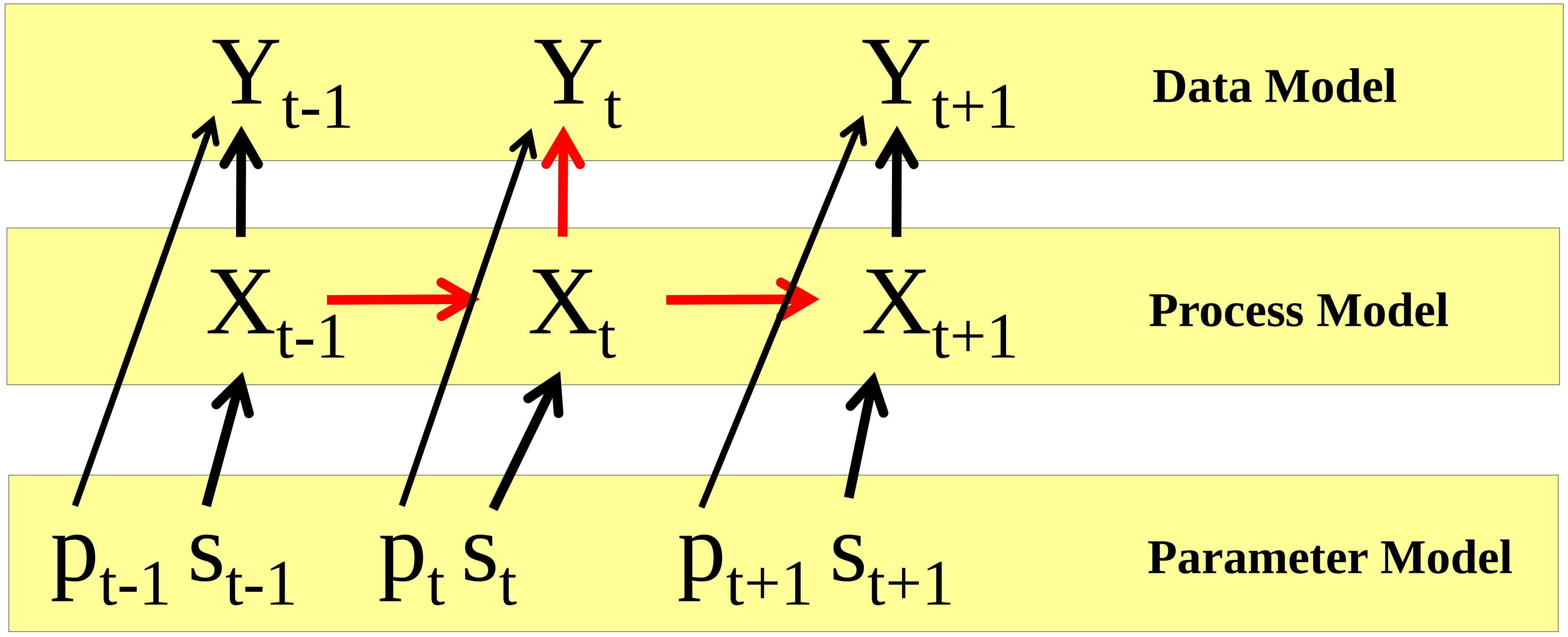


HM2 abundance
500.0
100.0
50.0
20.0
10.0
5.0
2.0
1.0
0.5
2013
2014
2015
2016
2017
2018
time

Capture-Recapture

- Individuals captured, marked, and released with goal of estimating population size.
- Over repeated censuses will recapture some fraction of the population
- Assume recapture is random

Mark Recapture State Space



Capture-Recapture

- Suppose an individual record consists of capture data

$$Y_i = [1,0,1,0,0]$$

- This is compatible with the following survival

$$X_i = [1,1,1,0,0]$$
$$X_i = [1,1,1,1,0]$$
$$X_i = [1,1,1,1,1]$$

Basic Mark-Recapture State Space

- Process model

$$P(X_t = 1 \mid X_{t-1} = 1) = s_t$$

$$P(X_t = 1 \mid X_{t-1} = 0) = 0$$

$$P(X_t = 0 \mid X_{t-1} = 1) = 1 - s_t$$

$$P(X_t = 0 \mid X_{t-1} = 0) = 1$$

Bernoulli Survival Probability

Basic Mark-Recapture State Space

- Process model

$P(X_t = 1 \mid X_{t-1} = 1) = s_t$

$P(X_t = 1 \mid X_{t-1} = 0) = 0$

$P(X_t = 0 \mid X_{t-1} = 1) = 1 - s_t$

$P(X_t = 0 \mid X_{t-1} = 0) = 1$

Bernoulli Survival Probability

$s^x (1-s)^{1-x}$

- Observation model

$P(Y_t = 1 \mid X_t = 1) = p_t$

$P(Y_t = 1 \mid X_t = 0) = 0$

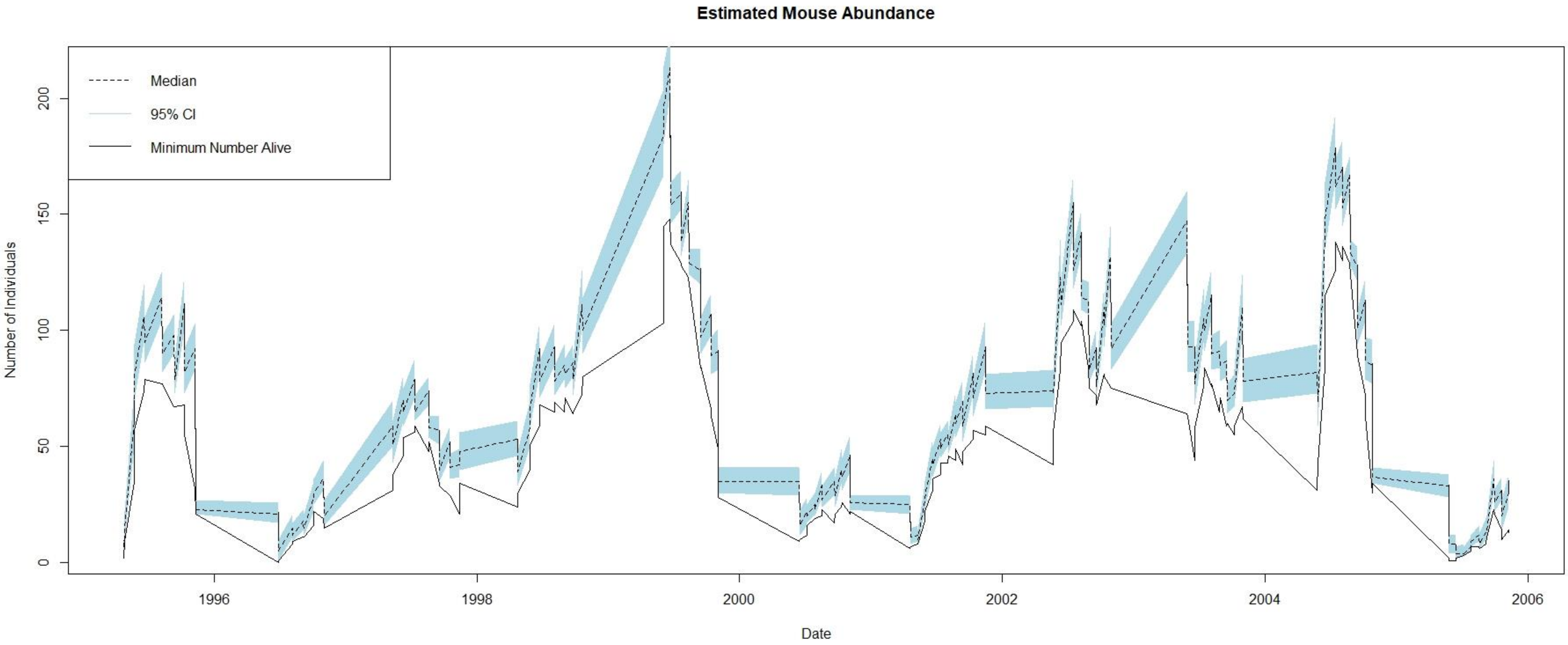
$P(Y_t = 0 \mid X_t = 0) = 1$

$P(Y_t = 0 \mid X_t = 1) = 1 - p_t$

Bernoulli Detection Probability

$p^x (1-p)^{1-x}$

- Priors on p and s (e.g. Beta)


Estimated Mouse Abundance
Median
95% CI
Minimum Number Alive
Number of Individuals
0
50
100
150
200
1996
1998
2000
2002
2004
2006
Date

John Foster

Generality of the State Space Model

- Neither X nor Y need be Normal
- X and Y don't need to be the same type of data
- X and Y don't need to have the same time scale
- Easily handles missing data (gaps)
- Can accommodate multiple data sources (Y's) – and they can be different data types